量子克隆之前世今生

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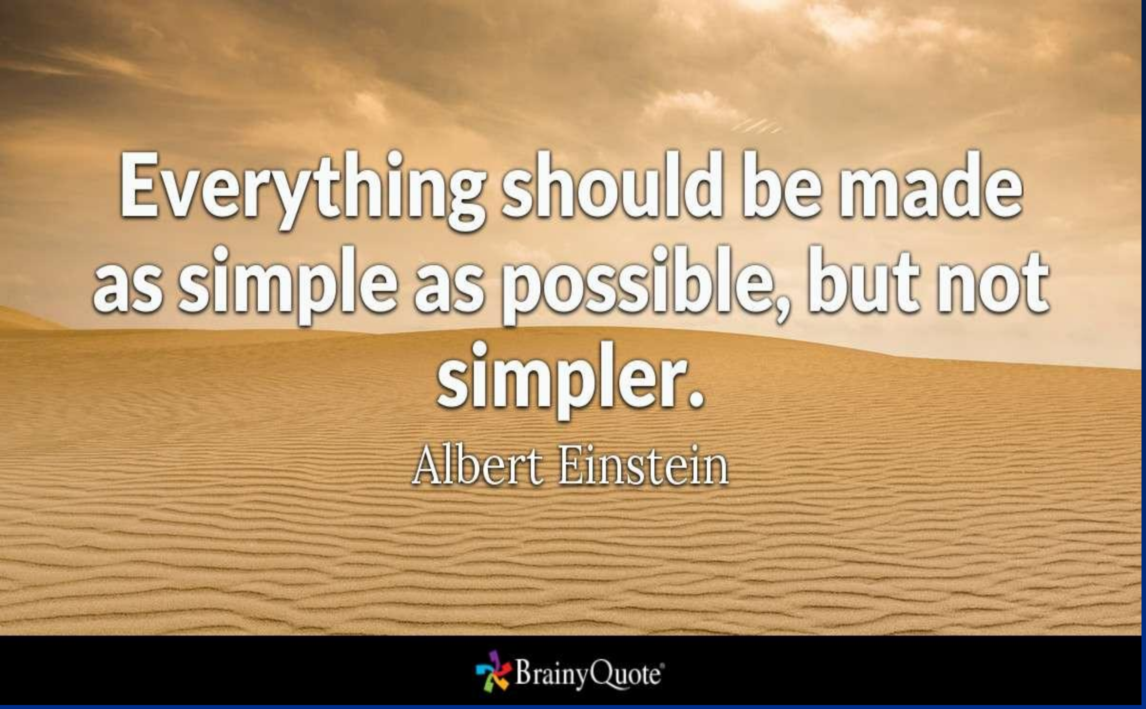


Make it simple,

because I can only understand

simple things.

--A. Peres

提纲

0. 引子

相对论: 光速不可逾越原理

量子论: Heisenberg 测不准原理

1. 量子克隆
2. 量子删除
3. 量子广播
4. 量子信息

0. 引子

让我们从两个

基本物理原理

开始.

相对论 (经典物理)

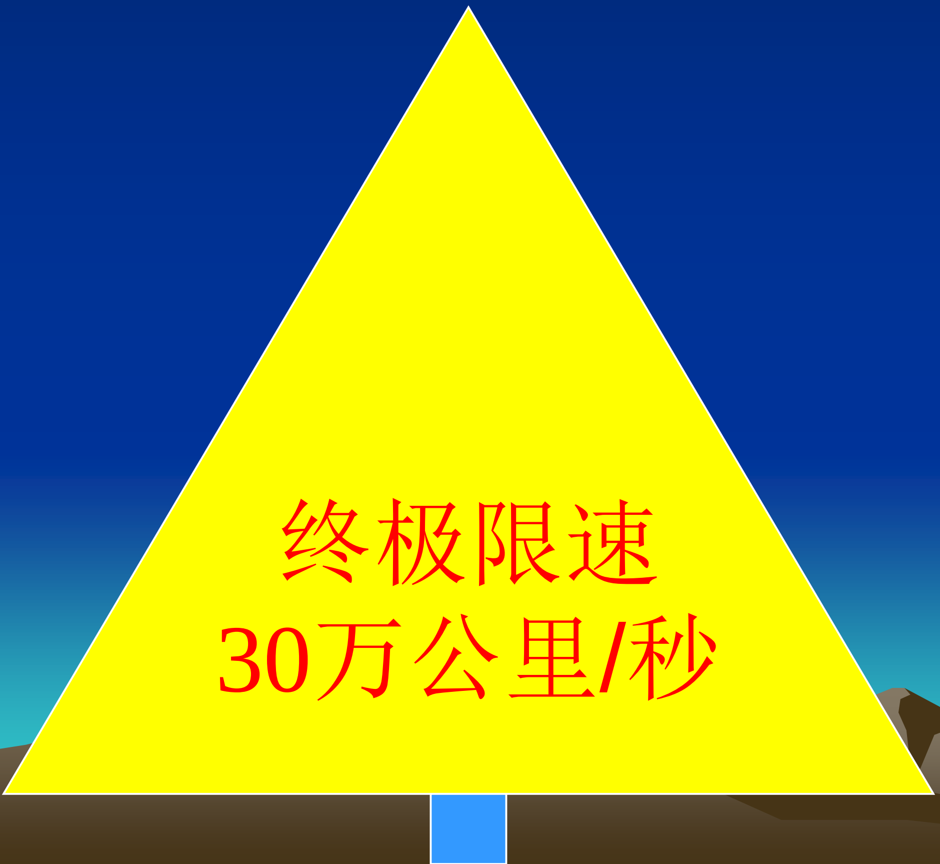
光速不可逾越原理

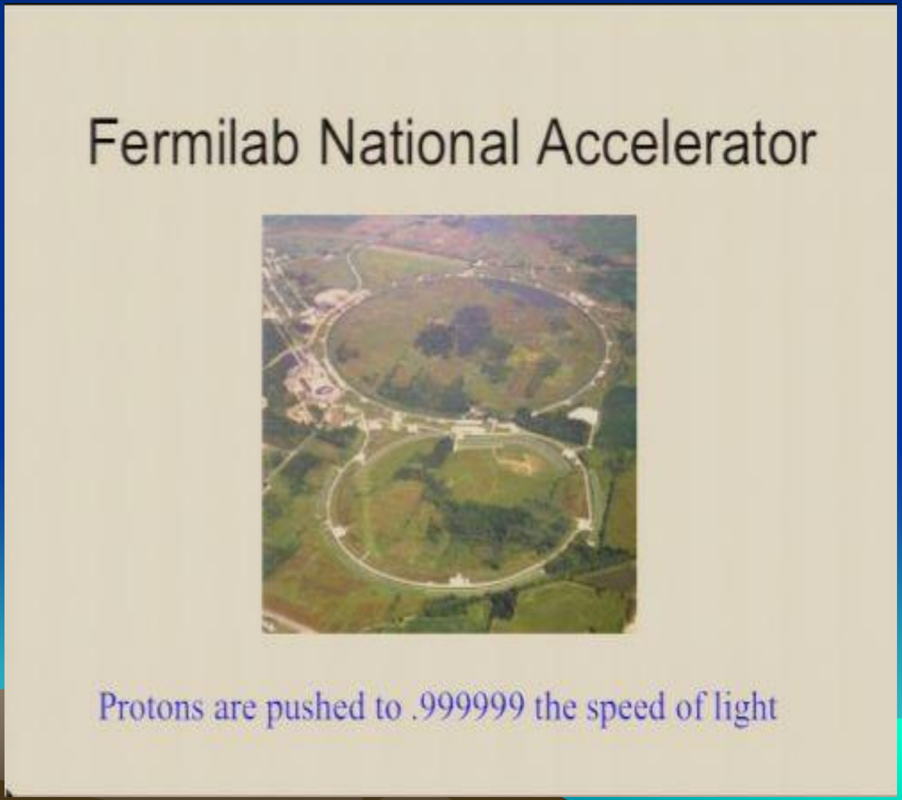
量子论 (现代物理)

Heisenberg测不准原理

Einstein 相对论：

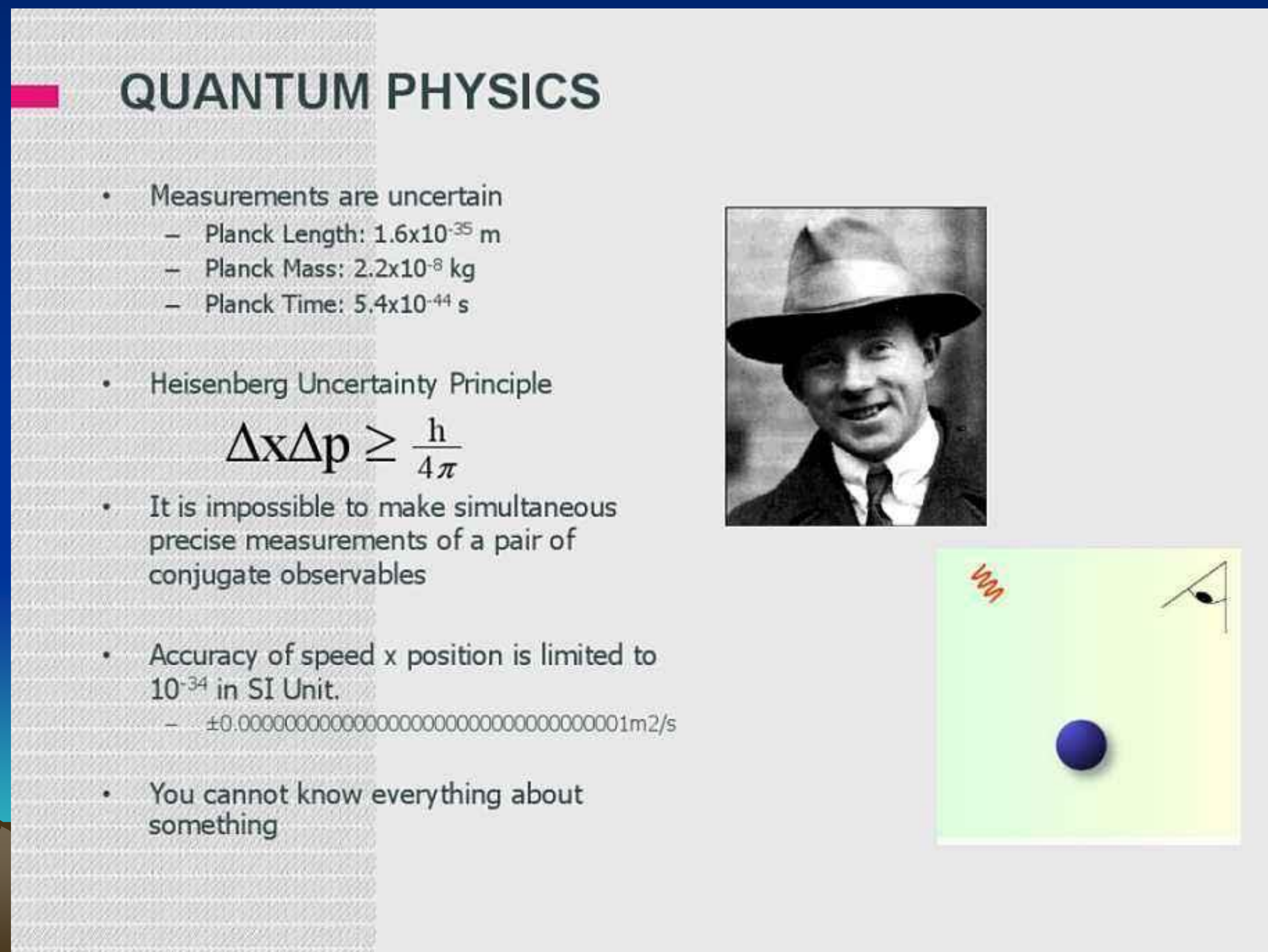
真空中光速 = 30万公里/ 秒，信息传递速度的上界





Heisenberg 量子力学：

测不准原理



相对论之前

牛顿

微积分, 力学, 光学,……

传奇:

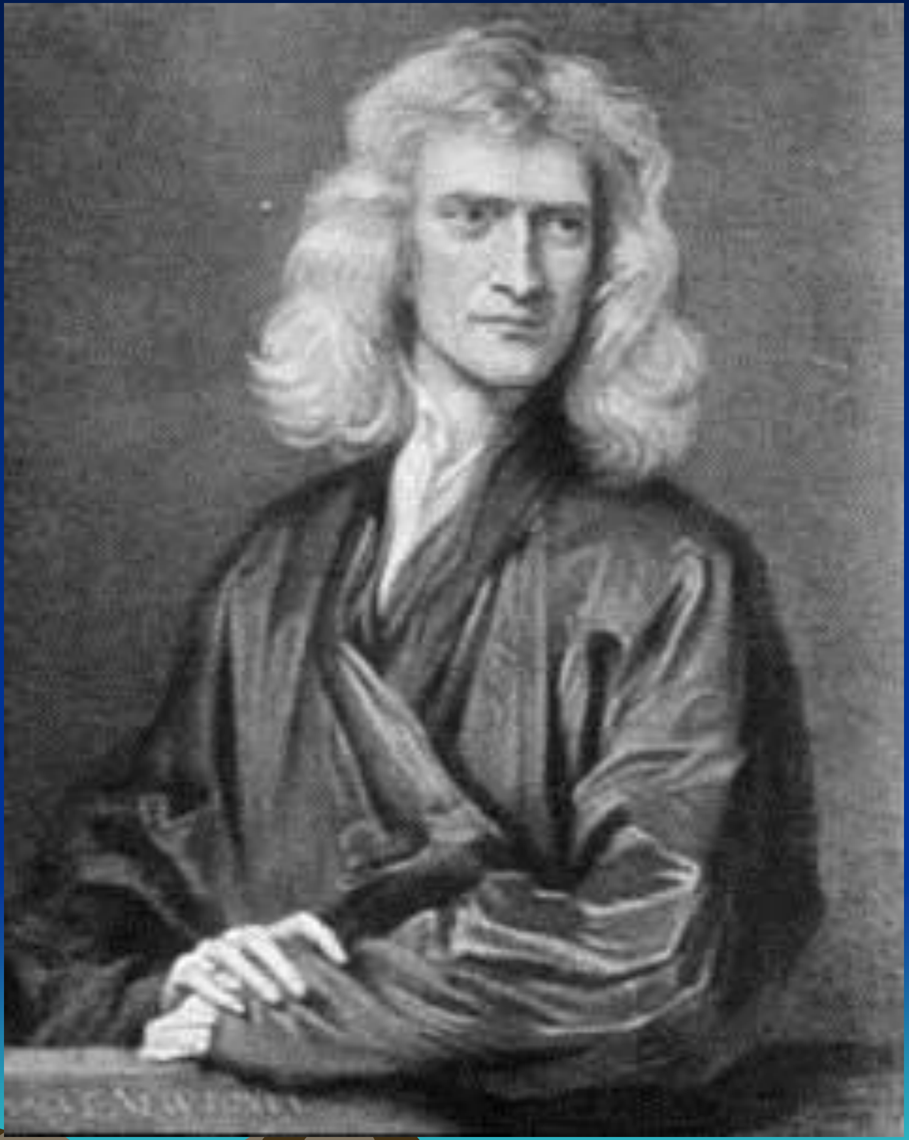
单身汉

国会议员

造币局局长

皇家学会主席

神秘的炼金术士



经典力学(牛顿力学)

第二定律：力 = 质量 X 加速度

加速度为轨道的二次微分.

基本出发点：状态 = (位置, 速度)

二阶运动方程的初始条件即为位置和速度.

对于牛顿来说, 宇宙是一个在创世之初由上帝紧了发条的巨大钟表, 从那时起就按照他的三大定律, 以完全精确预计的方式走向永恒.

经典力学的黯然无奈:

- 不能描述高速世界
- 不能描述微观运动

问题出在哪里?

牛顿说: 我不给空间, 时间, 地点和运动下定义, 这些

都是不言自明的!

相对论: (空间, 时间)是经验幻觉.

量子论: (地点, 运动)是经验幻觉.

Hilbert几何基础(The Foundations of Geometry)开篇

声明

设想有三组不同的对象：

第一组对象叫点

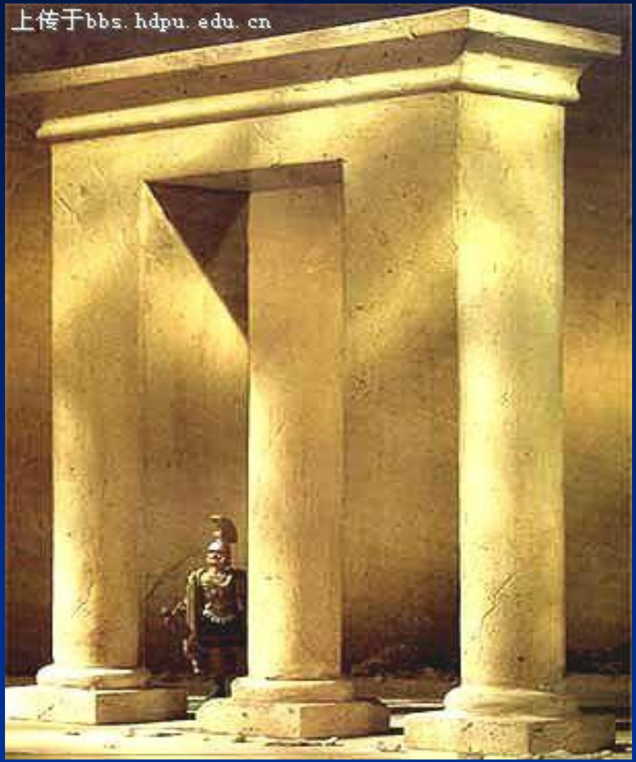
第二组对象叫线

第三组对象叫面

感觉与实在

空间、时间和物质，是人类认识的错觉. 宇宙中的存在只有场.

--爱因斯坦



在科学上,几乎每件事都是超过你直接经验的. 世间人往往仅以自己 的见闻和经验来评判事物,但他不知道，我们的感觉和经验经常在欺骗自己.

--威斯柯夫

为什么状态可由(位置, 速度)精确描述?

经典力学断言:

这是众所周知, 不言自明的.

量子力学妥协:

舍弃其一, 要么位置, 要么速度.

位置和速度(动量)是互为对偶，就像同一枚硬币的正反面.

什么是测不准原理?

……the position and velocity of an object cannot both be measured exactly at the same time, and that the concepts of exact position and exact velocity together have no meaning in nature.

--Britannica Concise Encyclopedia

Heisenberg先生驾车狂驶, 被警察截住.

警: 先生, 您超速了, 这是罚单.

H: 我在哪儿超速了?!

警: 那边测速点.

H: 荒唐! 根据我发现的的测不准原理,如果你知道任何东西的确切位置, 你就对他的速度一无所知!

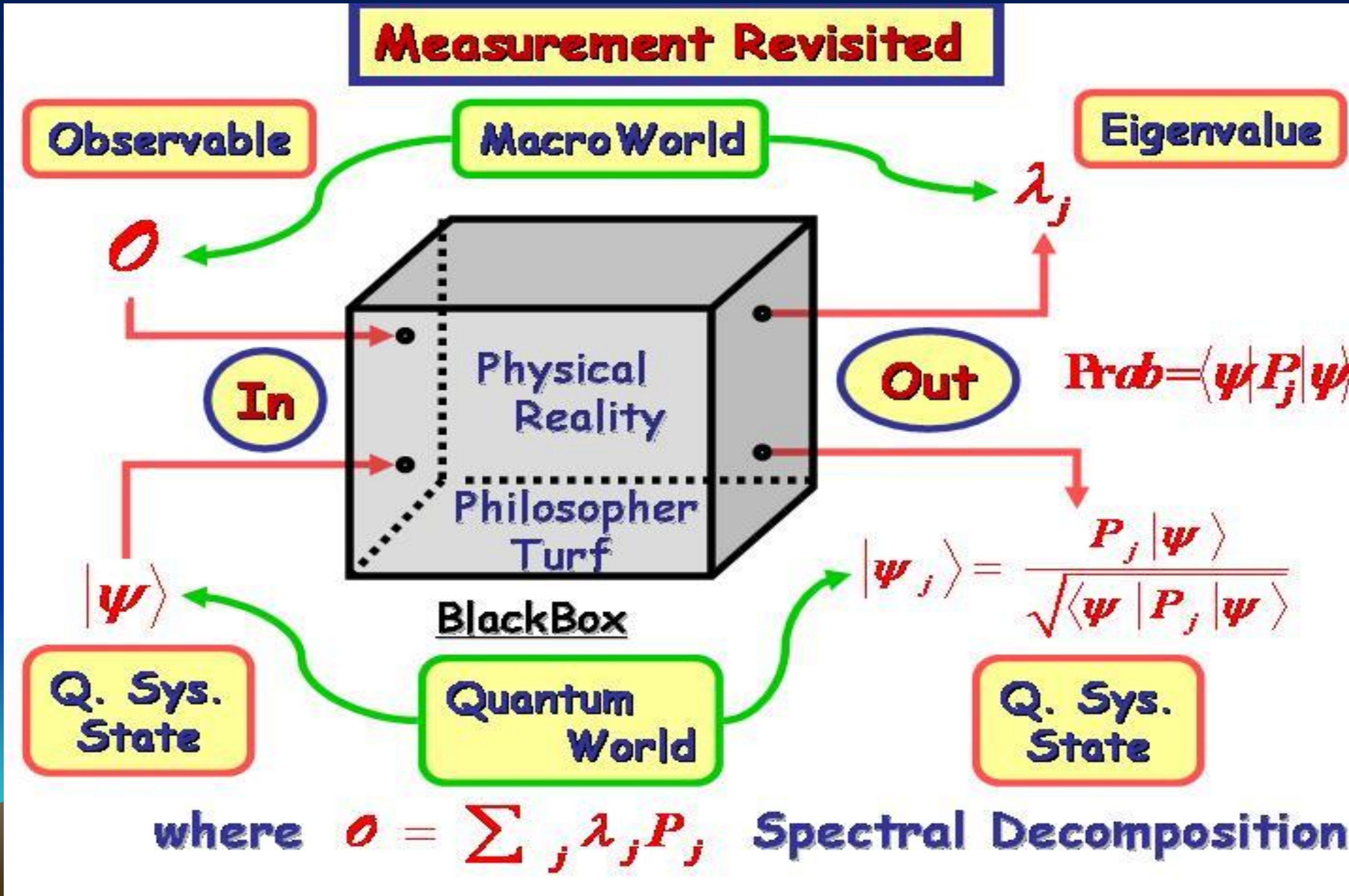
警: ???......%$*@#&...(抓狂?)

什么是量子测量

量子力学公理 (数学基础):

- 态: 由Hilbert 空间上的密度算子(自共轭，非负，迹为1) 表示
- 演化: Schrodinger 方程，Heisenberg方程，von Neumann-Landau 方程
- 测量：由正算子测度(resolution of identity)来描述

量子测量

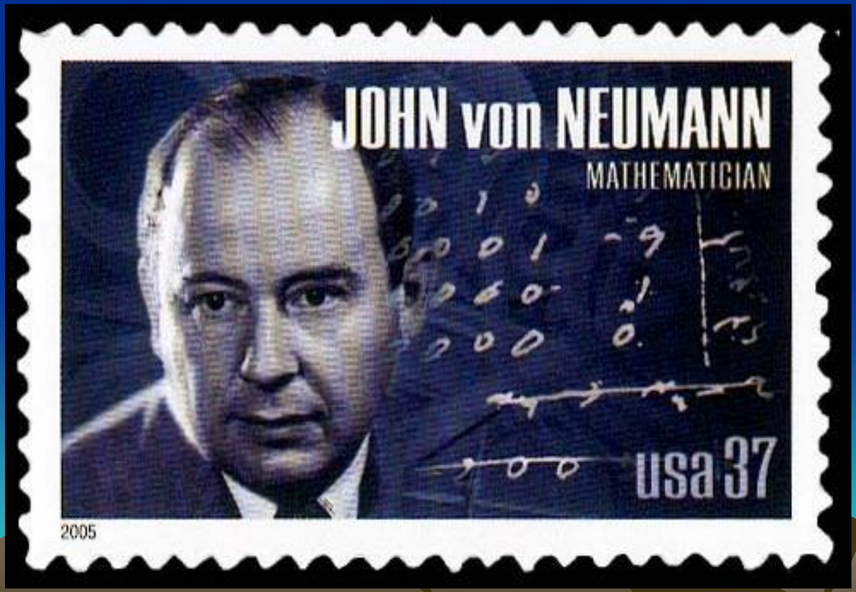


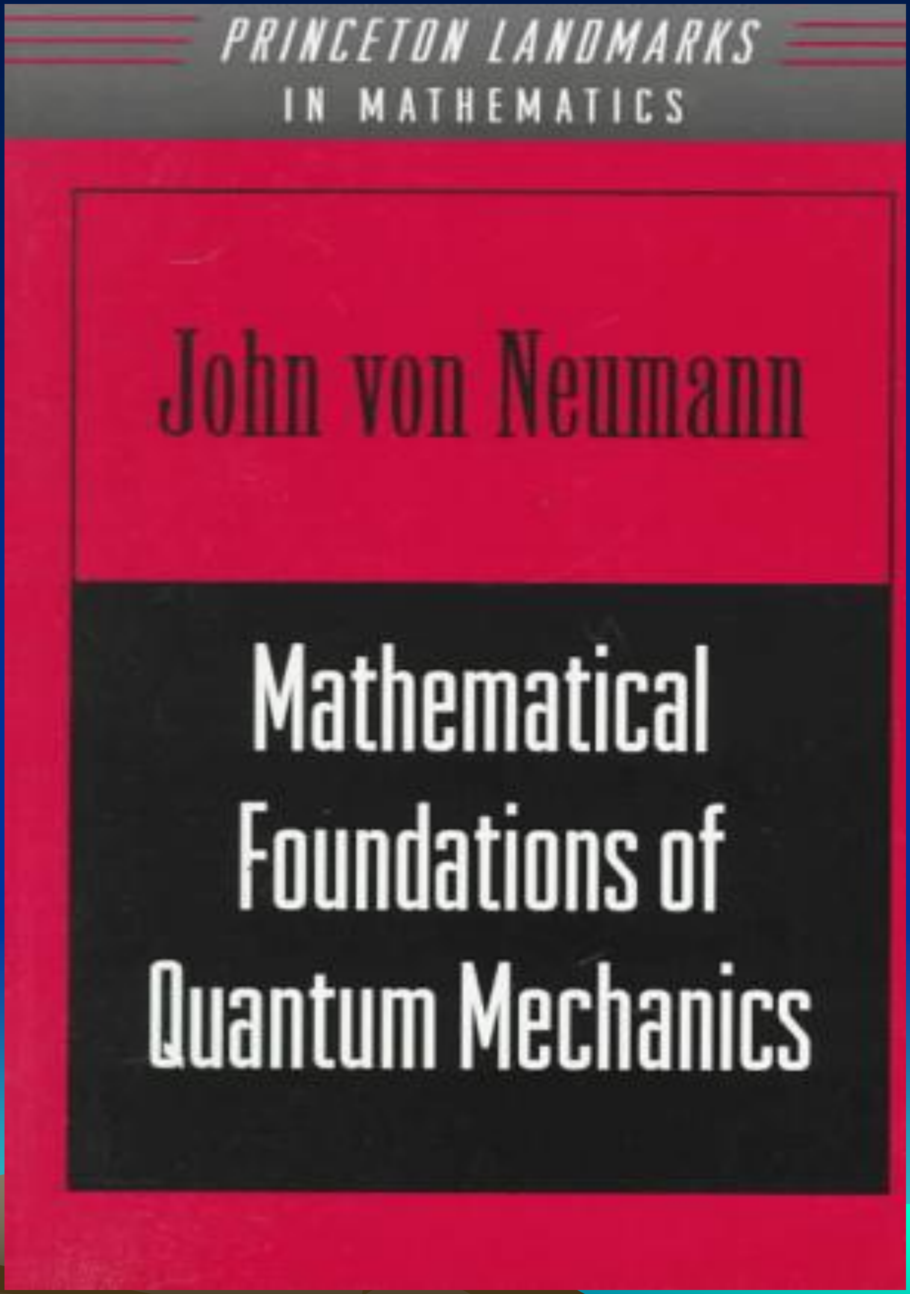
Measurement Revisited
Observable
Macro World
Eigenvalue
O
λ_j
Physical Reality
Philosopher Turf
In
Out
$Prob = \langle \psi | P_j | \psi \rangle$
$|\psi\rangle$
$|\psi_j\rangle = \frac{P_j |\psi\rangle}{\sqrt{\langle \psi | P_j | \psi \rangle}}$
BlackBox
Q. Sys. State
Quantum World
Q. Sys. State
where $O = \sum_j \lambda_j P_j$ Spectral Decomposition

量子力学的数学基础

von Neumann, 1932

Hilbert空间理论





量子测量问题

W. E. Lamb Jr. (1955, Nobel Prize)

Taught QM for more than 20 years.

On the beginning of the course, told the students,

``You must first learn the rules of calculation in quantum mechanics, then I will tell you about the theory of measurement…” Almost invariably, the time alloted to the course ran out before I had to fulfil my promise.

量子测量问题仍然是量子理论的一个核心未决问题.

1. Can the ``measurement problem" in quantum theory be resolved?
2. What does quantum information tell us about the nature of reality?

JOHN TEMPLETON FOUNDATION

Grant Opportunity Announcement

Quantum Physics and the Nature of Reality

Deadline for Inquiry: April 15, 2010

Quantum mechanics has been one of the most successful theories in science and is believed by many to underlie all known natural phenomena. More than eighty years after its discovery, however, a complete understanding of the theory's "fundamental principles" and implications still eludes us. Many believe that quantum physics poses a serious challenge to many cherished philosophical ideas and may present us with radically different visions of reality.

Quantum physics is currently going through a very fruitful period, partly due to the rapid development of the new field of quantum information. Many significant new ideas have been proposed, and a range of important new experiments are being performed. This is an opportune time to revisit the very foundations of quantum physics and to investigate the nature of reality.

The program "Quantum Physics and the Nature of Reality" is intended to support serious research on some of the most profound issues in science today. Proposals in theoretical physics, experimental physics, and the philosophy of quantum physics are welcome, as are interdisciplinary proposals, in response to either of the following Big Questions:

(1) Can the "measurement problem" in quantum physics be resolved?

(2) What does quantum information tell us about the nature of reality?

Budget range and term for individual projects: From $50,000 to $800,000 and for up to two years. Researchers in all areas of the physical sciences, philosophy of physics, and the related fields are encouraged to apply.

The selection process is overseen by an international jury (Chair: Anton Zeilinger).

For more information about the grant program, please visit: http://www.templeton.org/

For information on the application process, please visit: http://www.templeton.org/what_we_fund/our_grantmaking_process/

JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE - INVESTING IN THE BIG QUESTIONS

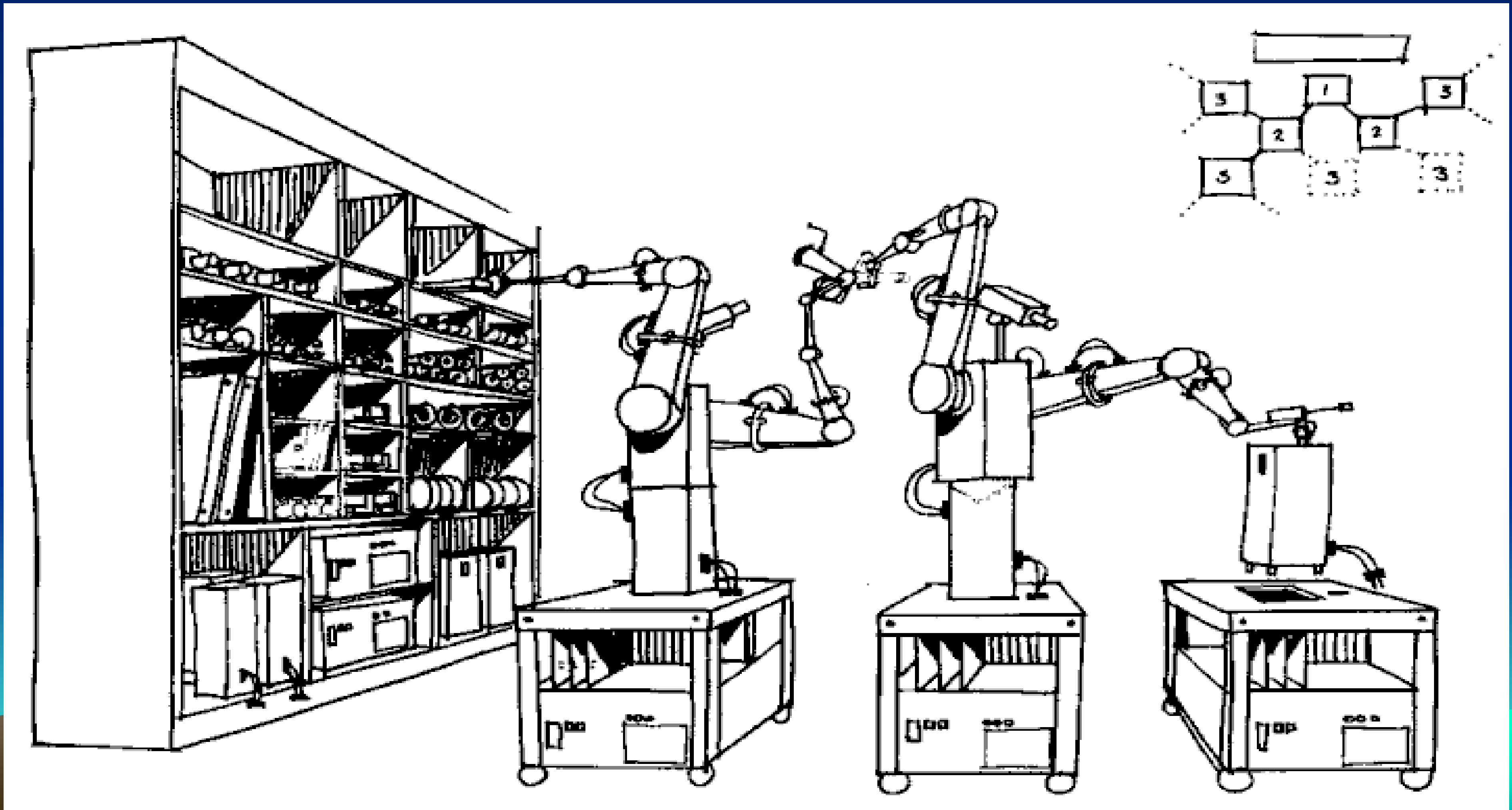
1. 量子克隆

克隆是生命的基础.

克隆是信息传播的手段.

经典世界里，信息是可克隆的.

Robot self-replication

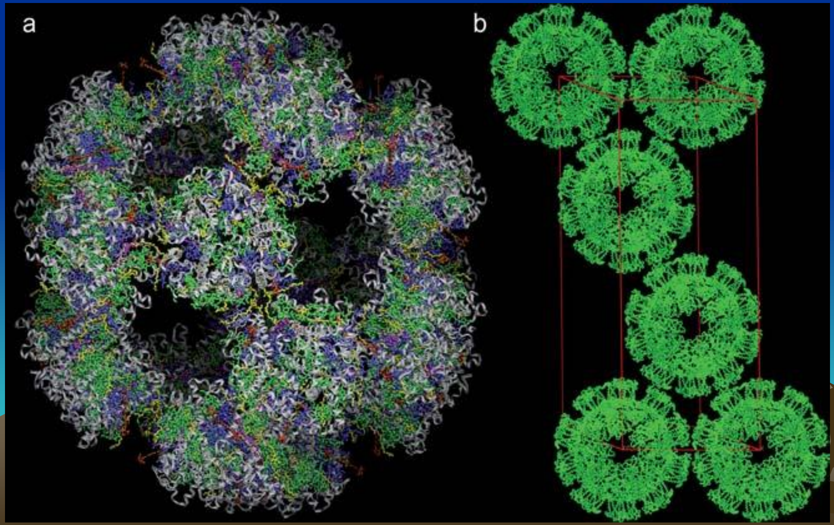


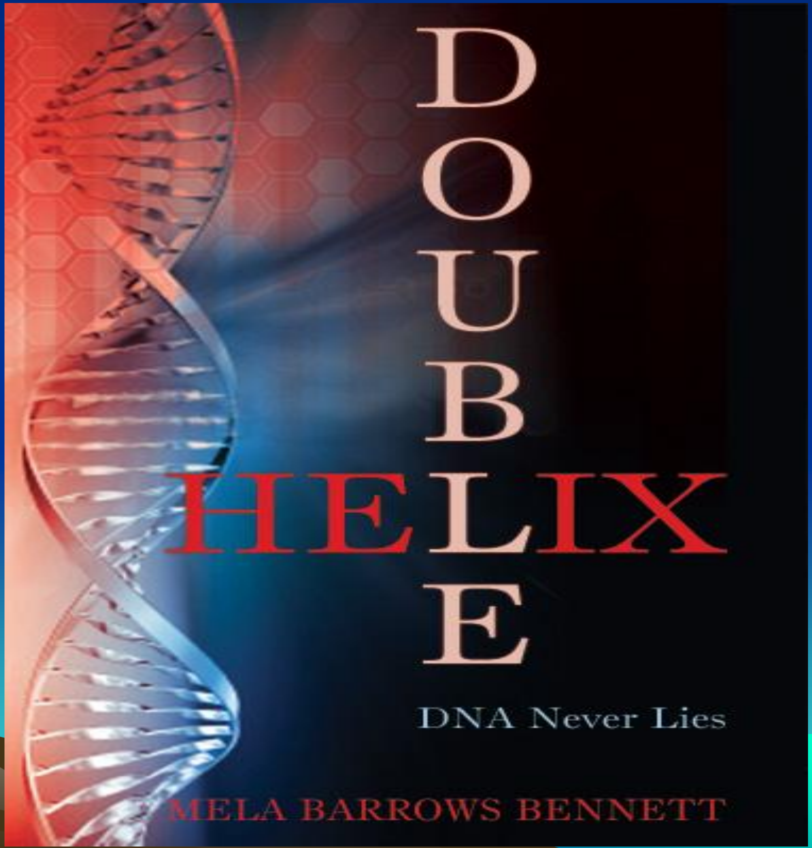
Self-growing lunar factory

信息与生物， DNA, 1953

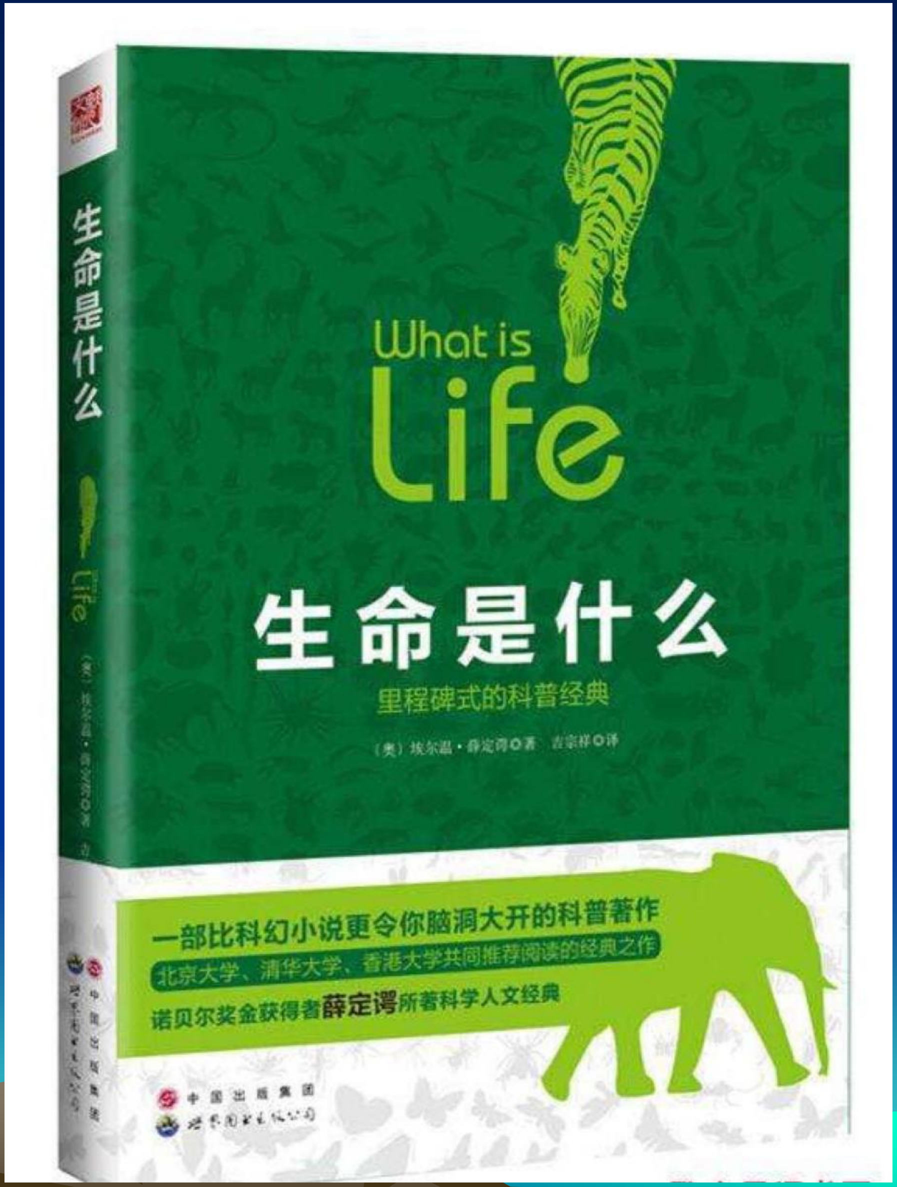
DNA 利用大分子来编码信息(50个原子/比特)，是生物克隆的基础. 这里虽然牵涉到量子系统， 但克隆的不是量子态. DNA大分子复制的只是部分信息，还有变异.

生物传存的是信息





E. Schrödinger



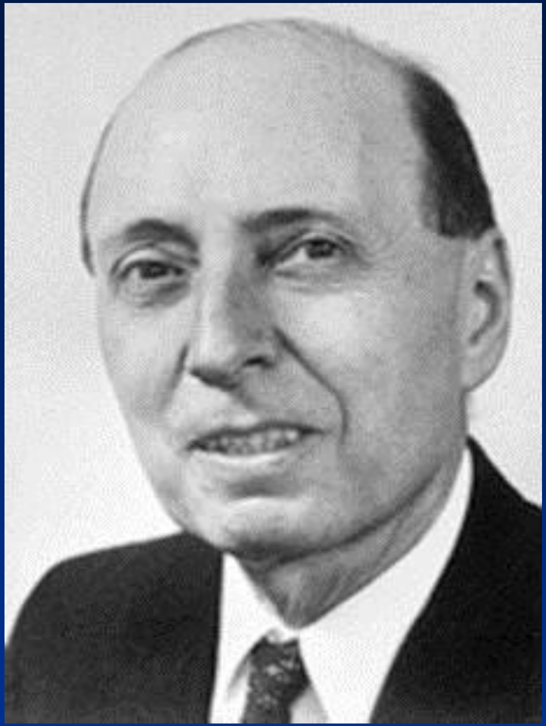
量子信息可克隆吗？

先回忆

几

件事.

故事 1，1967年



E. P. Wigner (1963, Nobel Prize)

The probability of the existence of self-reproducing unit

从量子力学基本原理计算自繁殖单位(例如生物)存在的概率, 发现其为零:

生物的存在与量子理论是矛盾的?

设H_1, H_2 是有限维 Hilbert 空间, $H = H_1 \otimes H_1 \otimes H_2$.

令 $R(H) = \{R \in U(H)$: 存在 $L \subset H_1$,

$0 \neq w \in H_1 \otimes H_2, R: L \otimes Cw \to L \otimes L \otimes H_2\}$.

若$\dim H_1 - \dim L$充分大， 则$R(H)$为$U(H)$中零测集.

直观地说，$L =$ 生物， $w =$ 奶粉

the chances are nil for the existence of a set of ``living” states for which one can find a nutrient of such nature that the interaction always leads to multiplication. -- Wigner

E. P. Wigner,

The unreasonable effectiveness of mathematics in the natural sciences,

Commun. Pure and Applied Math. 13, 1-14 (1960)

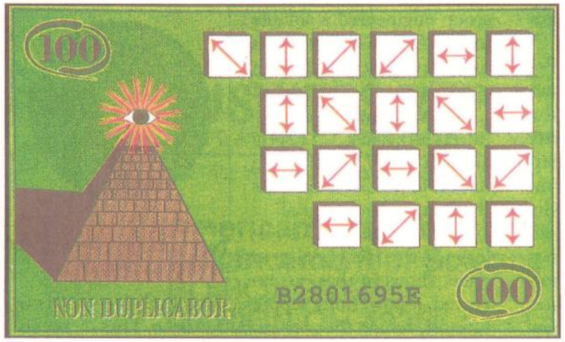
被引用次数 (Google Scholar)：2030

故事 2，1969年

Wisener 提出了量子钞票， 一种刻上若干量子态的无法伪造的钞票 (money that is physically impossible to counterfeit).

S. Wisener, Conjugate coding, ACM Signact News, 15 78-88 (1983)

Stephen Wiesner's idea for making quantum money, (circa 1970, published 1983).



In each bill, there is a sequence of quantum states in one of two complementary bases (so one of $|\updownarrow\rangle, |\leftrightarrow\rangle\, |\nearrow\rangle, |\nwarrow\rangle$). By the quantum no-cloning theorem, anyone who does not know the polarizations of these states cannot copy them.

Wisener 为Columbia大学研究生, 其父为MIT校长.

其文是量子密码的开山之作，被多家刊物拒稿，

十多年后, 即1983年才在Sigact发表.

Conjugate Coding *

Stephen Wiesner

Columbia University, New York, N.Y.
Department of Physics

The uncertainty principle imposes restrictions on the capacity of certain types of communication channels. This paper will show that in compensation for this "quantum noise", quantum mechanics allows us novel forms of coding without analogue in communication channels adequately described by classical physics.

故事 3，1981年

N. Herbut 提出了一种基于激光的

超光速传递信号的装置----Flash

First Laser-Amplified Superluminal Hookup

明知该文不对, 却将它投到 (奇怪?)

Foundations of Physics

还取了个耸人的名字： FLASH--A Superluminal Communicator Based upon a New Type of Quantum Measurement

Foundations of Physics 审稿人:

- A. Peres, 明知其错, 但不知错在何处,

推荐发表! (奇怪?)

- G. C. Ghirardi, 指出(初级)错误所在,

推荐拒稿!

Foundations of Physics, 1982

批准发表! (奇怪?)

Foundations of Physics, Vol. 12, No. 12, 1982

FLASH[1]—A Superluminal Communicator Based Upon a New Kind of Quantum Measurement

Nick Herbert[2]

Received January 15, 1982

The FLASH communicator consists of an apparatus which can distinguish between plane unpolarized (PUP) and circularly unpolarized (CUP) light plus a simple EPR arrangement. FLASH exploits the peculiar properties of "measurements of the Third Kind." One purpose of this article is to focus attention on the operation of idealized laser gain tubes at the one-photon limit.

1. INTRODUCTION

The theorem of Bell guarantees that two quantum systems which have interacted in the past can no longer be regarded as independent systems.[1] The mathematical inseparability of the quantum theoretical representation is an essential part of nature, not a mere accident of the formalism. These once interacting systems—which in general may be space-like separated, hence truly isolated according to special relativity—remain in some sense connected in a manner unmediated, unmitigated, and immediate. If this instant quantum connection were directly observable—rather than indirectly verified via Bell's argument—it would put quantum mechanics into conflict with special relativity by permitting faster than light signaling.

Can Quantum Connectedness Act as a Medium for Superluminal Communication?

This question has been considered by physicists at Berkeley[2] and Trieste,[3] and answered in the negative. A typical scheme imagines systems

[1] FLASH: acronym for First Laser-Amplified Superluminal Hookup.
[2] Box 261, Boulder Creek, California.

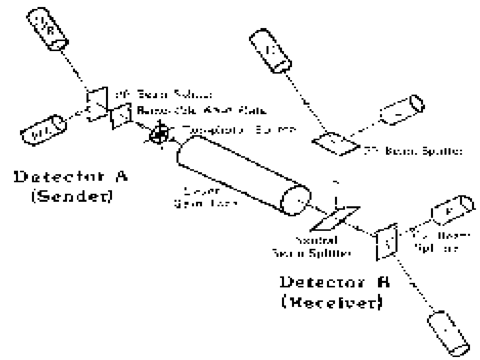


Fig. 1. The FLASH detection process. Photons in beam B (traveling to the right) are rendered either circularly unpolarized (CUP) or plane unpolarized (PUP) by positioning of the quarter-wave plate in beam A (traveling to the left). Each B photon is amplified by a multichannel laser gain tube and the resulting unpolarized burst of light is examined for counting asymmetry in either the CP or PP channel.

polarization can now be measured with a beam-splitter arrangement. Fig. 1 illustrates one polarimeter possibility.

Imagine that a plane polarized (H) photon has been detected at A. This means that a plane polarized (V) photon is incident on the B subsystem. This photon is amplified by the gain tube into N V polarized photons which are separated by the neutral beam splitter into two packets of roughly $N/2$ photons each, all plane polarized in the vertical plane. One of these subbeams is directed to a CP splitter where it is divided equally into R and L beams, and detected. The other subbeam is directed to a PP splitter and all $N/2$ photons are deflected into the V detector. The signature of a vertically polarized photon is:

H: 0 photons

V: $N/2$ photons

R: $N/4$ photons

L: $N/4$ photons

In a similar manner RCP and LCP photons leave a unique signature. It is

错误的论文激发人们 (Ghirardi除外) 去找出其错误所在:

W. K. Wootters and W. H. Zurek,

A single quantum cannot be cloned,

Nature, 299, 802 (1982)

Manuscript received 11 Aug. 1982

被引用次数：4383

A single quantum cannot be cloned

W. K. Wootters*

Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712, USA

W. H. Zurek

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If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6–8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \to |A_s\rangle|ss\rangle \qquad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \to |A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle \qquad (2)$$

and

$$|A_0\rangle|\leftrightarrow\rangle \to |A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (3)$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha=\beta=2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle) \to \alpha|A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle+\beta|A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (4)$$

If the apparatus states $|A_{\text{vert}}\rangle$ and $|A_{\text{hor}}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle+\beta|\leftrightarrow\leftrightarrow\rangle \qquad (5)$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^{+}_{\text{vert}}+\beta a^{+}_{\text{hor}})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle+2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle+\beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

* Present address: Department of Physics and Astronomy, Williams College, Williamstown, Massachusetts 01267, USA.

Milonni (unpublished work) has shown that the process of stimulated emission does not lead to quantum amplification, because if there is stimulated emission there must also be—with equal probability in the case of one incoming photon—spontaneous emission, and the polarization of a spontaneously emitted photon is entirely independent of the polarization of the original.

It is conceivable that a more sophisticated amplifying apparatus could get around Milonni's argument. We have therefore presented the above simple argument, based on the linearity of quantum mechanics, to show that no apparatus, however complicated, can amplify an arbitrary polarization.

We stress that the question of replicating individual photons is of practical interest. It is obviously closely related to the quantum limits on the noise in amplifiers[10,11]. Moreover, an experiment devised to establish the extent to which polarization of single photons can be replicated through the process of stimulated emission is under way (A. Gozzini, personal communication; and see ref. 12). The quantum mechanical prediction is quite definite; for each perfect clone there is also one randomly polarized, spontaneously emitted, photon.

We thank Alain Aspect, Carl Caves, Ron Dickman, Ted Jacobson, Peter Milonni, Marlan Scully, Pierre Meystre, Don Page and John Archibald Wheeler for enjoyable and stimulating discussions.

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1. Born, M. & Wolf, E. *Principles of Optics* 4th edn (Pergamon, New York, 1970).
2. Herbert, N. *Found. Phys.* (in the press).
3. Einstein, A., Podolsky, B. & Rosen, N. *Phys. Rev.* **47**, 777–780 (1935).
4. Bohm, D. *Quantum Theory*, 611–623 (Prentice-Hall, Englewood Cliffs, 1951).
5. Kocher, C. A. & Commins, E. D. *Phys. Rev. Lett.* **18**, 575–578 (1967).
6. Freedman, S. J. & Clauser, J. R. *Phys. Rev. Lett.* **28**, 938–941 (1972).
7. Fry, E. S. & Thompson, R. C. *Phys. Rev. Lett.* **37**, 465–468 (1976).
8. Aspect, A., Grangier, P. & Roger, G. *Phys. Rev. Lett.* **47**, 460–463 (1981).
9. Bussey, P. J. *Phys. Lett.* **90A**, 9–12 (1982).
10. Haus, H. A. & Mullen, J. A. *Phys. Rev.* **128**, 2407–2410 (1962).
11. Caves, C. M. *Phys. Rev.* **D15**, (in the press).
12. Gozzini, A. *Proc. Symp. on Wave-Particle Dualism* (eds Diner, S., Fargue, D., Lochak, G. & Selleri, F) (Reidel, Dordrecht, in the press).

D. Dieks,

Communications by EPR devices,

Physics Letters 92 A , 271 (1982).

Manuscript received 17 Aug. 1982

Volume 92A, number 6 PHYSICS LETTERS 22 November 1982

COMMUNICATION BY EPR DEVICES

D. DIEKS

Fysisch Laboratorium, Rijksuniversiteit Utrecht, Utrecht, The Netherlands

Received 17 August 1982
Revised manuscript received 21 September 1982

A recent proposal to achieve faster-than-light communication by means of an EPR-type experimental set-up is examined. We demonstrate that such superluminal communication is not possible. The crucial role of the linearity of the quantum mechanical evolution laws in preventing causal anomalies is stressed.

P. W. Milonni and L. Hardies,

Photons cannot always be replicated,

Physics Letters, 92 A, 371 (1982).

Manuscript received 5 Aug. 1982

PHOTONS CANNOT ALWAYS BE REPLICATED

P.W. MILONNI [1]
Theoretical Division (T-12), Los Alamos National Laboratory, University of California, Los Alamos, NM 87545, USA

and

M.L. HARDIES
Department of Physics, University of Arkansas, Fayetteville, AR 72701, USA

Received 5 August 1982
Revised manuscript received 1 October 1982

Perfect and certain replication of any single photon is impossible.

L. Mandel,

Is a photon amplifier always polarization dependent?

Nature, 304, 188 (1983).

实际构造了第一个最优克隆机器.

Is a photon amplifier always polarization dependent?

WITH the help of an ingeniously simple argument, Wootters and Zurek[1] have drawn attention to the fact that there exists no amplifying apparatus such as one or more excited atoms, for example, which will 'clone' an incident photon of arbitrary polarization. More precisely, if $|1_{\varepsilon_1}\rangle$ is a one-photon state of polarization characterized by some complex unit vector ε_1, a photon amplifier cannot always turn this into the state $|2_{\varepsilon_1}\rangle$ for an arbitrary ε_1. In general, the two-photon state will be some superposition of states $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ and $|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle$, where ε_1, ε_2 are orthogonal unit polarization vectors, or even a mixture of states. However, the conclusion of Wootters and Zurek should not be misinterpreted to mean that the output of a photon amplifier has to be polarization dependent.

If the amplifier is in the form of an excited two-level atom in the state $|+\rangle$, with transition dipole moment μ, then the amplitude of the two-photon state $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ depends on the scalar product $\mu\cdot\varepsilon_1^*$, and would even vanish if the dipole moment were orthogonal to the polarization of the incoming photon. In these conditions there would be no stimulated emission at all, only spontaneous emission. This is apparent if we write for the final state after a short interaction time Δt in the interaction picture

$$|\Psi_{\rm final}\rangle = \exp(-i\hat{H}_{\rm I}\Delta t/\hbar)|1_{\varepsilon_1}, 0_{\varepsilon_2}\rangle|+\rangle \quad (1)$$

with an electric dipole interaction

$$\hat{H}_{\rm I} = g\sum_{s=1}^{2}[\mu\cdot\varepsilon_s^*\hat{\sigma}^{(-)}\hat{a}_s^\dagger + hc] \quad (2)$$

We have limited ourselves to a Hilbert space (where the operators are distinguished by the caret ˆ) with just two resonant plane wave modes, and have written $\hat{\sigma}^{(-)}$ and $\hat{a}_s$ for the atomic and field lowering operators. From equations (1) and (2) one finds immediately, after tracing over atomic variables, that after a short time Δt the resulting two-photon state is of the form

$$|\Phi\rangle = \frac{\sqrt{2}\mu\cdot\varepsilon_1^*|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle + \mu\cdot\varepsilon_2^*|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle}{(2|\mu\cdot\varepsilon_1^*|^2 + |\mu\cdot\varepsilon_2^*|^2)^{1/2}} \quad (3)$$

The first term is attributable to stimulated emission and the second to spontaneous emission into the other mode. Clearly $|\Phi\rangle$ becomes $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ only when the dipole moment μ is parallel to the polarization ε_1, and it becomes $|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle$ when μ is orthogonal to ε_1. In other words, for this simple one-atom amplifier the final state depends on the polarization of the incoming state, as Wootters and Zurek have pointed out.

However, lest it be thought that it is the sensitivity to polarization that is the essential element in preventing cloning of the incident photon, we now show that it is not difficult, at least in principle, to construct an amplifier whose output is independent of the polarization. For this purpose we consider a system of two resonant, excited atoms with orthogonal transition dipole moments $\mu_a = |\mu|\varepsilon_a$, $\mu_b = |\mu|\varepsilon_b$, where ε_a, ε_b are complex, orthogonal unit polarization vectors. We will not go into the non-trivial question how such a state can be produced in practice, but it might perhaps be done by exposing the atoms separately to different light beams and then bringing them together. The atoms are assumed to be sufficiently close that they experience the same field. Then the interaction may be taken to be of the form

$$\hat{H}_{\rm I} = g\sum_{s=1}^{2}(\hat{\sigma}_a^{(-)}\mu_a + \hat{\sigma}_b^{(-)}\mu_b)\cdot\varepsilon_s^*\hat{a}_s^\dagger + hc \quad (4)$$

and equation (1) leads to the following (unnormalized) two-photon state

$$|-_a, +_b\rangle[\sqrt{2}\mu_a\cdot\varepsilon_1^*|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle + \mu_a\cdot\varepsilon_2^*|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle] + |+_a, -_b\rangle[\sqrt{2}\mu_b\cdot\varepsilon_1^*|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle + \mu_b\cdot\varepsilon_2^*|1\varepsilon_1, 1\varepsilon_2\rangle] \quad (5)$$

After tracing over atomic variables we encounter a mixed two-photon state, with density operator

$$\hat{\rho} = \tfrac{2}{3}|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle\langle 2_{\varepsilon_1}, 0_{\varepsilon_2}| + \tfrac{1}{3}|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle\langle 1_{\varepsilon_1}, 1_{\varepsilon_2}| \quad (6)$$

This is independent of the polarization of the incident photon and of the two atomic transition dipole moments, so long as they are orthogonal. The first term evidently corresponds to stimulated emission, and the state $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ is twice as probable as $|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle$, which is attributable to spontaneous emission. There is no cloning, and the general conclusion of Wootters and Zurek[1] is, of course, borne out. But the essential element that prevents cloning is here seen to be the spontaneous emission, rather than any dependence of amplifier gain on polarization. A similar conclusion was also reached in another connection by Milonni and Hardies[2].

This work was supported in part by the NSF.

L. MANDEL

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA

1. Wootters, W. K. & Zurek, W. H. *Nature* **299**, 802 (1982).
2. Milonni, P. W. & Hardies, M. L. *Phys. Lett.* **92A**, 321 (1982).

On replicating photons

WOOTTERS AND ZUREK[1] have recently considered whether it is possible to build a quantum mechanical device which will simply duplicate an arbitrarily polarized incoming photon. They consider two possible situations. In the first, the final state of the device depends on the polarization of the photon. In this case, a photon beam of arbitrary polarization will give rise to a mixed, rather than a pure, final state and will therefore not be properly replicated.

In the second situation the final state of the replicator is considered to be independent of the photon polarization. The authors (as also in a subsequent paper by Dieks[2]) demonstrate an inconsistency in the quantum mechanical description of this situation which leads them to conclude that in it, too, photon replication is impossible to achieve. However, this second situation is unphysical for a rather serious reason: if the final state of the replicator is independent of the photon polarization, then angular momentum conservation is violated. Photons of different polarization are in different spin states (or different linear combinations of spin states). Thus the polarization of the emitted photon must affect the final angular momentum state of the replicator which emits it.

P. J. BUSSEY

Department of Natural Philosophy, University of Glasgow, Glasgow G12 8QQ, UK

1. Wootters, W. K. & Zurek, W. H. *Nature* **299**, 802–803 (1982).
2. Dieks, D. *Phys. Lett.* **92A**, 271 (1982).

WOOTTERS AND ZUREK REPLY—Bussey points out that if the amplifier's final state were independent of polarization, then angular momentum would not be conserved. The question of angular momentum conservation is, however, more subtle than it may seem at first, as the following example shows. (This example is related to work of Wigner, Araki and Yanase on the limitations imposed by conservation laws on the accuracy of measurements[1-4].)

Let $|l\rangle$ be a certain state of the amplifier which is an eigenstate of L_z with eigenvalue l, L_z being the component of angular momentum along the direction of motion of the photon. Assume that when a right- or left-handed circularly polarized photon interacts with the amplifier in this state, the following angular-momentum-conserving transformation occurs:

$$|\text{one right-handed photon}\rangle\otimes|l\rangle \rightarrow |\text{two right-handed photons}\rangle\otimes|l-1\rangle$$

2002, Peres公开为自己推荐发表Herbut的错误文章辩解，其得意之情溢于言表:

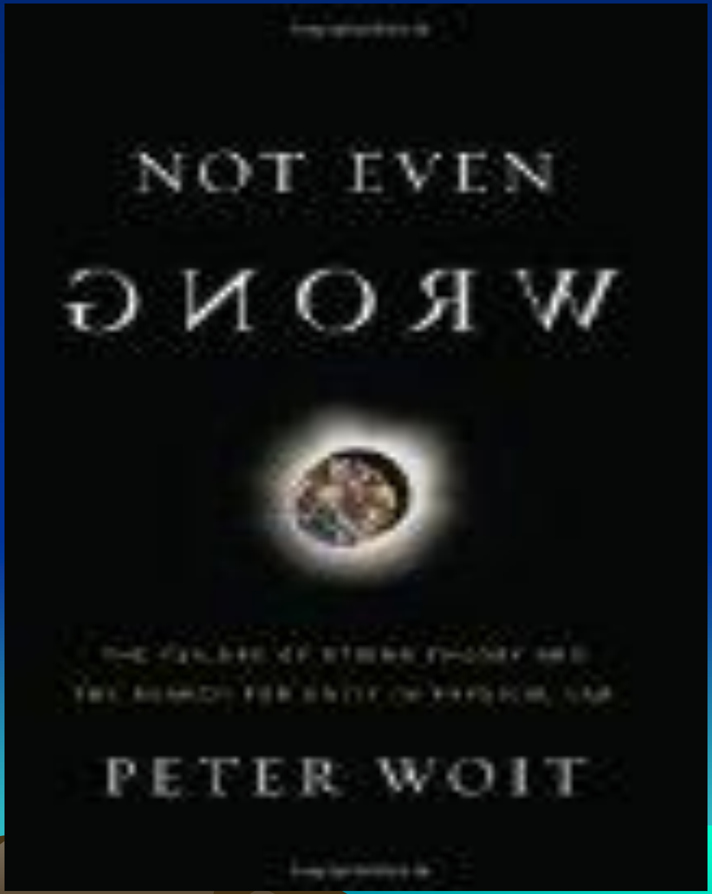
Nick Herbert's erroneous paper was a spark that generated immense progress. There also are many wrong papers that have been published in reputable journals, some of them by renowned scientists. Their bad influence may last for years. For these, I decline all responsibility.

--A. Peres, How the No-Cloning Theorem Got its name, *Fortschritte der Physik*. 51 (45), 458–461 (2003)

错误并不可怕，或并不一定是坏事

可怕的是连错误都不是！

It's not even wrong (Pauli).



Peres 是何许人？

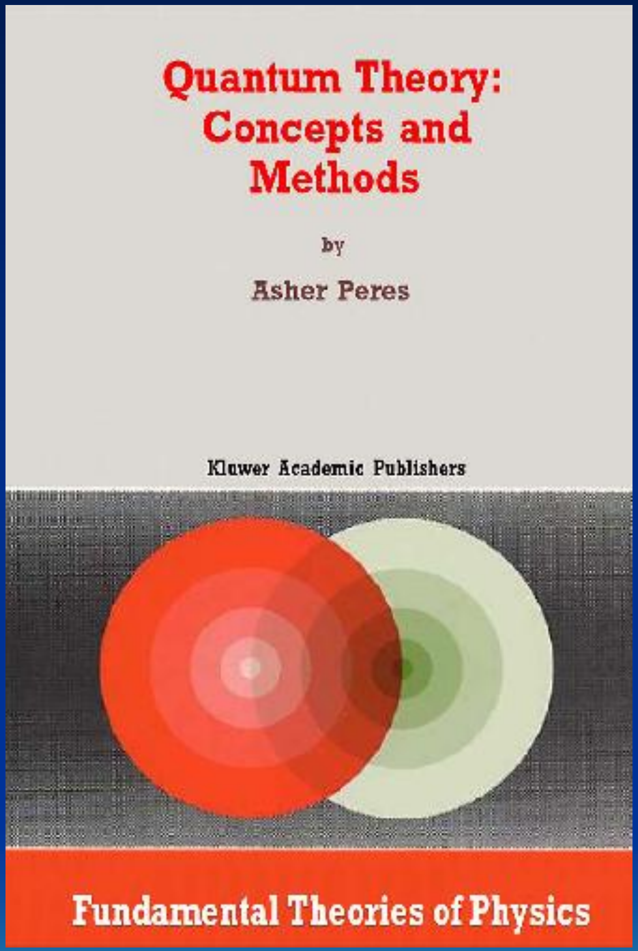
A. Peres (1945-2005)

obtained Ph.D. in 1959 at

Technion–Israel Institute of Technology

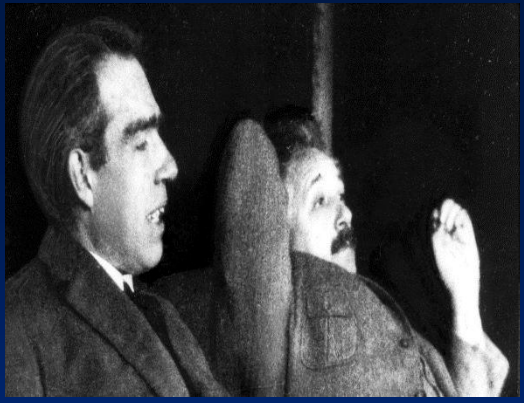
under Nathan Rosen

EPR: Einstein, Podolsky, Rosen



EPR: Einstein 对量子力学的诘难

Einstein-Bohr 论战: 位置-动量 Reality



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, Phys. Rev. Lett. 70, 1895 (1993) .

(top, left) Richard Jozsa, William K. Wootters, Charles H. Bennett. (bottom, left) Gilles Brassard, Claude Crépeau, Asher Peres. Photo: André Berthiaume.

纠缠的判别

Peres 判据

Positive Partial Transpose

被引用次数：4361

PHYSICAL REVIEW LETTERS

VOLUME 77 | 19 AUGUST 1996 | NUMBER 8

Separability Criterion for Density Matrices

Asher Peres*

Department of Physics, Technion–Israel Institute of Technology, 32 000, Haifa, Israel

(Received 8 April 1996)

A quantum system consisting of two subsystems is *separable* if its density matrix can be written as $\rho = \sum_A w_A \rho'_A \otimes \rho''_A$, where ρ'_A and ρ''_A are density matrices for the two subsystems, and the positive weights w_A satisfy $\sum w_A = 1$. In this Letter, it is proved that a necessary condition for separability is that a matrix, obtained by partial transposition of ρ, has only non-negative eigenvalues. Some examples show that this criterion is more sensitive than Bell's inequality for detecting quantum inseparability. [S0031-9007(96)00911-8]

PACS numbers: 03.65.Bz, 03.65.Ca

A striking quantum phenomenon is the inseparability of composite quantum systems. Its most famous example is the violation of Bell's inequality, which may be detected if two distant observers, who independently *measure* subsystems of a composite quantum system, *report* their results to a common site where that information is analyzed [1]. However, even if Bell's inequality is satisfied by a given composite quantum system, there is no guarantee that its state can be *prepared* by two distant observers who receive *instructions* from a common source. For this to be possible, the density matrix ρ has to be separable into a sum of direct products,

$$\rho = \sum_A w_A \rho'_A \otimes \rho''_A, \qquad (1)$$

where the positive weights w_A satisfy $\sum w_A = 1$, and where ρ'_A and ρ''_A are density matrices for the two subsystems. A separable system always satisfies Bell's inequality, but the converse is not necessarily true [2–5]. In this Letter, I shall derive a simple algebraic test, which is a *necessary* condition for the existence of the decomposition (1). I shall then give some examples showing that this criterion is more restrictive than Bell's inequality, or than the α-entropy inequality [6].

The derivation of this separability condition is best done by writing the density matrix elements explicitly, with all their indices [1]. For example, Eq. (1) becomes

$$\rho_{m\mu,n\nu} = \sum_A w_A (\rho'_A)_{mn} (\rho''_A)_{\mu\nu}. \qquad (2)$$

Latin indices refer to the first subsystem, Greek indices to the second one (the subsystems may have different dimensions). Note that this equation can always be satisfied if we replace the quantum density matrices by classical Liouville functions (and the discrete indices are replaced by canonical variables **p** and **q**). The reason is that the only constraint that a Liouville function has to satisfy is being non-negative. On the other hand, we want quantum density matrices to have non-negative *eigenvalues,* rather than non-negative elements, and the latter condition is more difficult to satisfy.

Let us now define a new matrix,

$$\sigma_{m\mu,n\nu} \equiv \rho_{n\mu,m\nu}. \qquad (3)$$

The Latin indices of ρ have been transposed, but not the Greek ones. This is not a unitary transformation but, nevertheless, the σ matrix is Hermitian. When Eq. (1) is valid, we have

$$\sigma = \sum_A w_A (\rho'_A)^T \otimes \rho''_A. \qquad (4)$$

Since the transposed matrices $(\rho'_A)^T \equiv (\rho'_A)^*$ are non-negative matrices with unit trace, they can also be legitimate density matrices. It follows that *none of*

Herbut 是何许人?

物理出身

打工为生

嬉皮士(hippies)

Fundamental Fysiks Group 成员

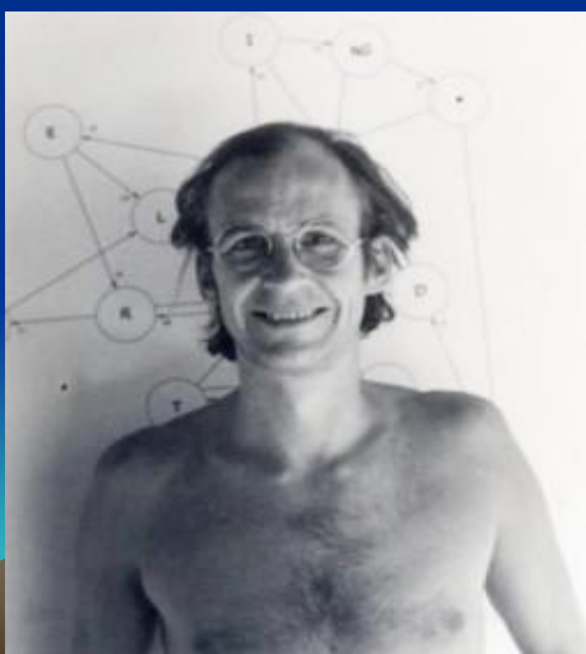


FIGURE I.1. The "new physicists" as counterculture darlings. *Left* (standing, left to right): Jack Sarfatti, Saul-Paul Sirag, Nick Herbert; (kneeling) Fred Alan Wolf, ca. 1975. *Right*: Jack Sarfatti as the eccentric genius of North Beach, 1979. (*Left*, courtesy Fred Alan Wolf; *right*, photograph by Robert L. Jone, courtesy Robert L. Jones and Jack Sarfatti.)

量子力学的创始者如爱因斯坦、玻尔、海森堡、玻恩和薛定谔等人在创立量子力学的过程中有大量关于基本概念和佯谬的讨论，并且经常上升到哲学的高度.

直到二战，学生在学习量子力学时也同样会花相当多的时间在这些概念问题上.

然而二战后，美国物理学界的主流对这些哲学问题丧失了兴趣，典型态度是

“快闭嘴，去计算！”(shut up and calculate)

例如，L. Shiff 的教科书

Quantum Mechanics (有李淑娴、陈崇光译的中文版)

是这一时期美国大学广泛采用的教材. 该书删去了此前量子力学教科书中大量的关于基本概念的解释和哲学的长篇讨论，却加入了大量难度相当高的技术问题.

嬉皮士(hippies)

上世纪 60 年代，

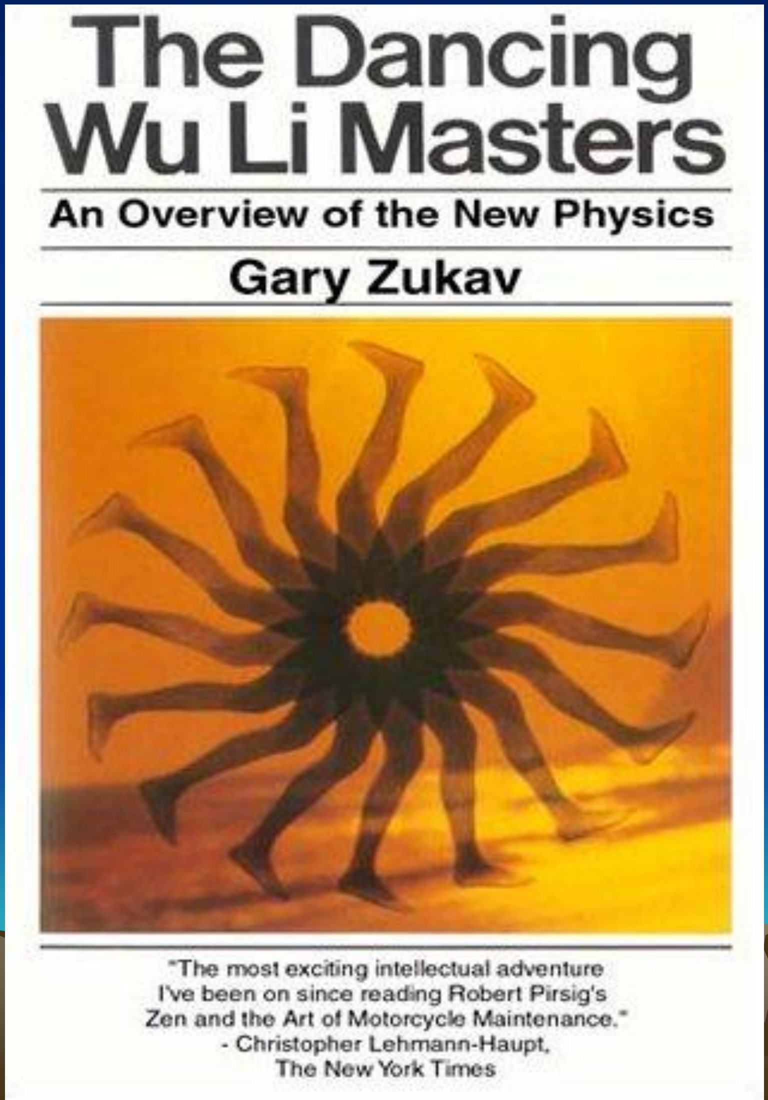
西方出现的一批反抗当时习俗和政治的年轻人，

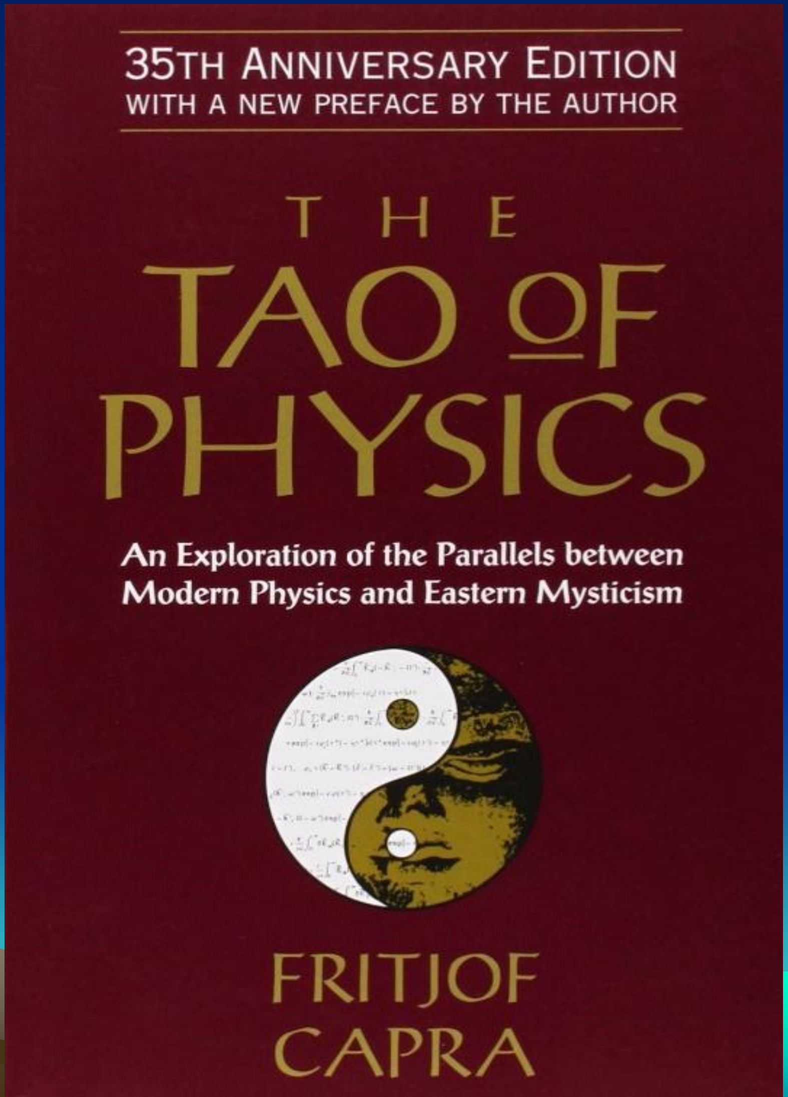
他们无意于传统的生活和工作，

而要探索自己的路.

Fundamental Fysiks Group (嬉皮士讨论班)

其思想种子后来发芽了，其中一个就是量子信息学





嬉皮士如何拯救了物理

科学、

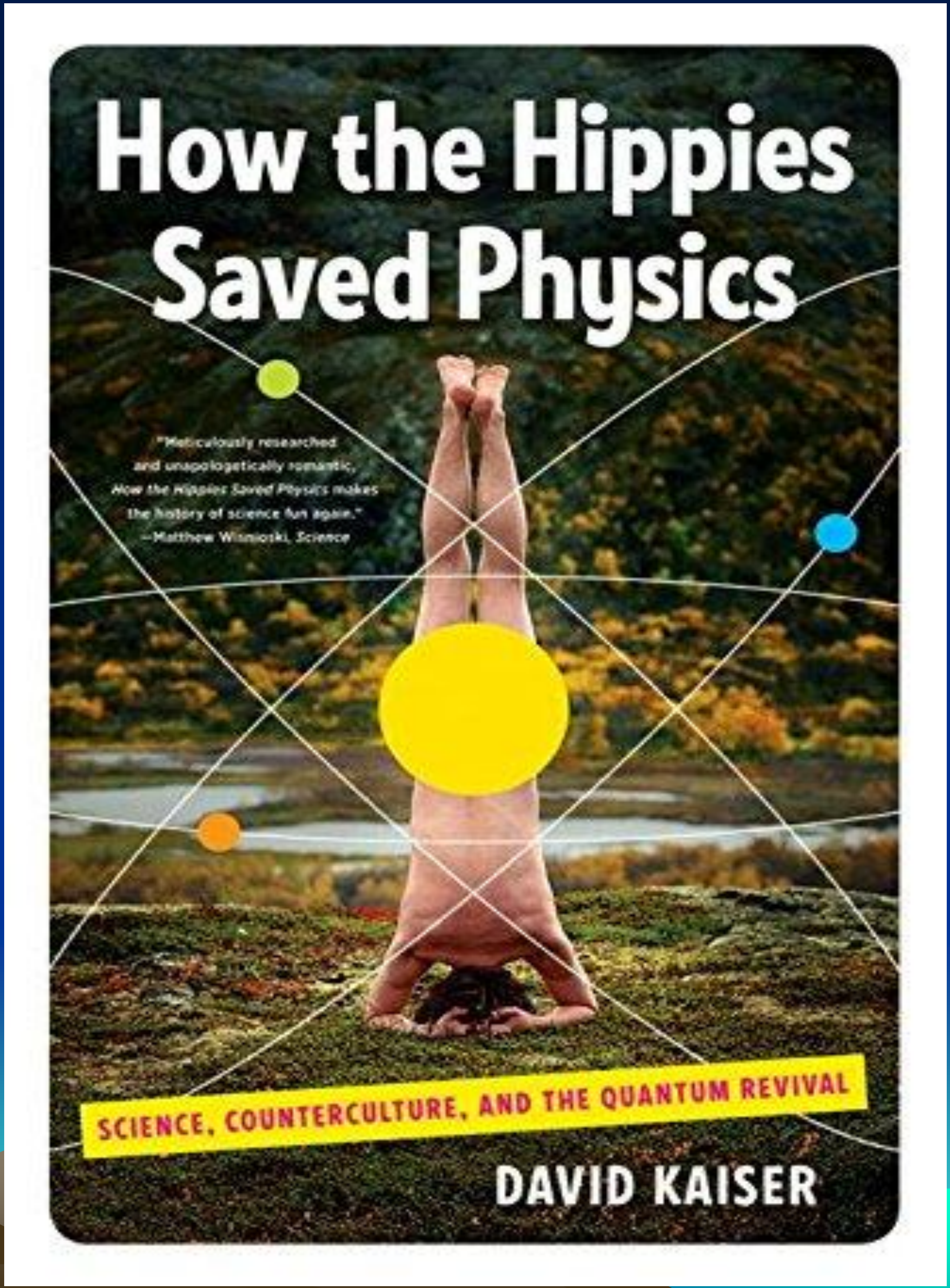
反主流文化

和量子复兴

D. Kaiser

MIT 物理教授

科学史系主任



量子不可克隆定理产生了巨大而深远的影响.....

Google Scholar搜索 quantum cloning

获得约 78,000 条结果

理论 (物理意义， 数学刻画)

实验

应用

量子不可克隆定理是量子理论中

基本,

深刻,

而又简单

的原理!

既然又重要, 又简单, 何以 在量子理论诞生 (1900) 后

80 多年，在其成熟 (1920s) 后半个多时纪，才由偶然的

事件催生?!

失去的机会

- Einstein, *A*-*B* 系数
- Von Neumann, 自繁殖机
- Towens, 激光
- Wigner，生物在量子条件下产生的概率
- Wiesner, 量子钞票
- Park, 量子跃迁
- Ghirardi, 审稿

Einstein

受激辐射不能单独存在, 总是伴随有自发辐射.

否则， 可以用受激辐射来克隆光子的状态.

受激辐射与自发辐射的比率正好使得最优渐近克隆无法满足利用量子非局域性进行超光速通讯.

Einstein 首次发现受激辐射与自发辐射的比率关系，这个关系正好可由他如此厌恶的量子非局域性(不能进行超光速通讯)导出!

von Neumann

Lectures delivered in 1948 and 1949:

conceptual proposal for a physical non-biological self-replicating system

The Theory of Self-Replicating Automata， Univ. Illinois Press, 1966 (work by von Neumann in 1952)

Self-replicator based on cellular automata

Towens (1964年Nobel 物理奖), 1957年

其关于Maser的 phenomenological方程

描述中隐含渐近克隆的最优保真度.

C. H. Towens, How the Laser Happened, Oxford, 2002

K. Shimoda, H. Takahasi, C. H. Townes, Fluctuations in amplification of quanta with application to maser amplifiers, J. Phys. Soc. Japan, 12, 686-700 (1957)

Wigner, 1967年

The probability of the existence of a self-reproducing unit

量子测量问题

Wisener， 1970

在其开创量子密码的文章中, 已实际应用量子不可克隆定理. 可惜超越了时代十多年, 无人搭理.

Conjugate Coding *

Stephen Wiesner

Columbia University, New York, N.Y.
Department of Physics

The uncertainty principle imposes restrictions on the capacity of certain types of communication channels. This paper will show that in compensation for this "quantum noise", quantum mechanics allows us novel forms of coding without analogue in communication channels adequately described by classical physics.

J. L. Park,

The concept of transition in quantum mechanics,

Foundations of Physics, 1, 23 (1970)

文章反主流！

但已实际证明

量子不可克隆定理,

可惜超越了时代十

多年，被视而不见.

To construct a measurement procedure, we conventionally adopt some initial state for the apparatus, then seek a correlation-producing interaction that converts the apparatus **M** into a new state which embodies information about the initial state of the system **S**. Specifically, we take α as the initial state for **M**.

To devise a nondisturbing measurement scheme, a unitary evolution operator T must be found that effects the following state evolution for $\mathbf{S} + \mathbf{M}$:

$$T\psi\alpha = \psi\psi \tag{1}$$

where ψ is the initial state of **S**. Such an interaction, if it exists, transfers the state specification of **S** to **M**, yet **S** emerges in the same state it was in at the beginning of the measurement. Hence, measurements upon **M** yield measurement results for **S** without changing the state of **S**.

It is convenient to divide the question as to the existence of T into two parts:

(s) Is there a T independent of ψ which satisfies (1), i.e., can a *simple* nondisturbing measurement be performed?

(h) Can a T be found for any specific ψ which satisfies (1), i.e., can an *historical* nondisturbing measurement be performed?

The answer to (s) turns out to be negative, as the following argument demonstrates. Let $a \equiv \langle\alpha, \psi\rangle$, $b \equiv \langle\beta, \psi\rangle$, so that $\psi = a\alpha + b\beta$. A *simple* nondisturbing T must satisfy

具讽刺或纳闷意味的是，Peres, Wootters 都引用过Park的那篇文章：

- C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, Phys. Rev. Lett. **70**, 1895 (1993)

  被引用次数(Google Scholar)：12044

- A. Peres, Neumark's theorem and quantum inseparability, Foundations of Physics, 20, 1441–1453 (1990)

Ghirardi的郁闷：

没重视 Herbert 的错误？没早点公开指出其错误所在？

Nuovo Cimento 78B, 9 (1983).

Journal of Physics A 40, 2891 (2007).

IOP Publishing Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. 40 (2007) 2891–2917 doi: 10.1088/1751-8113/40/12/S01

Some reflections inspired by my research activity in quantum mechanics

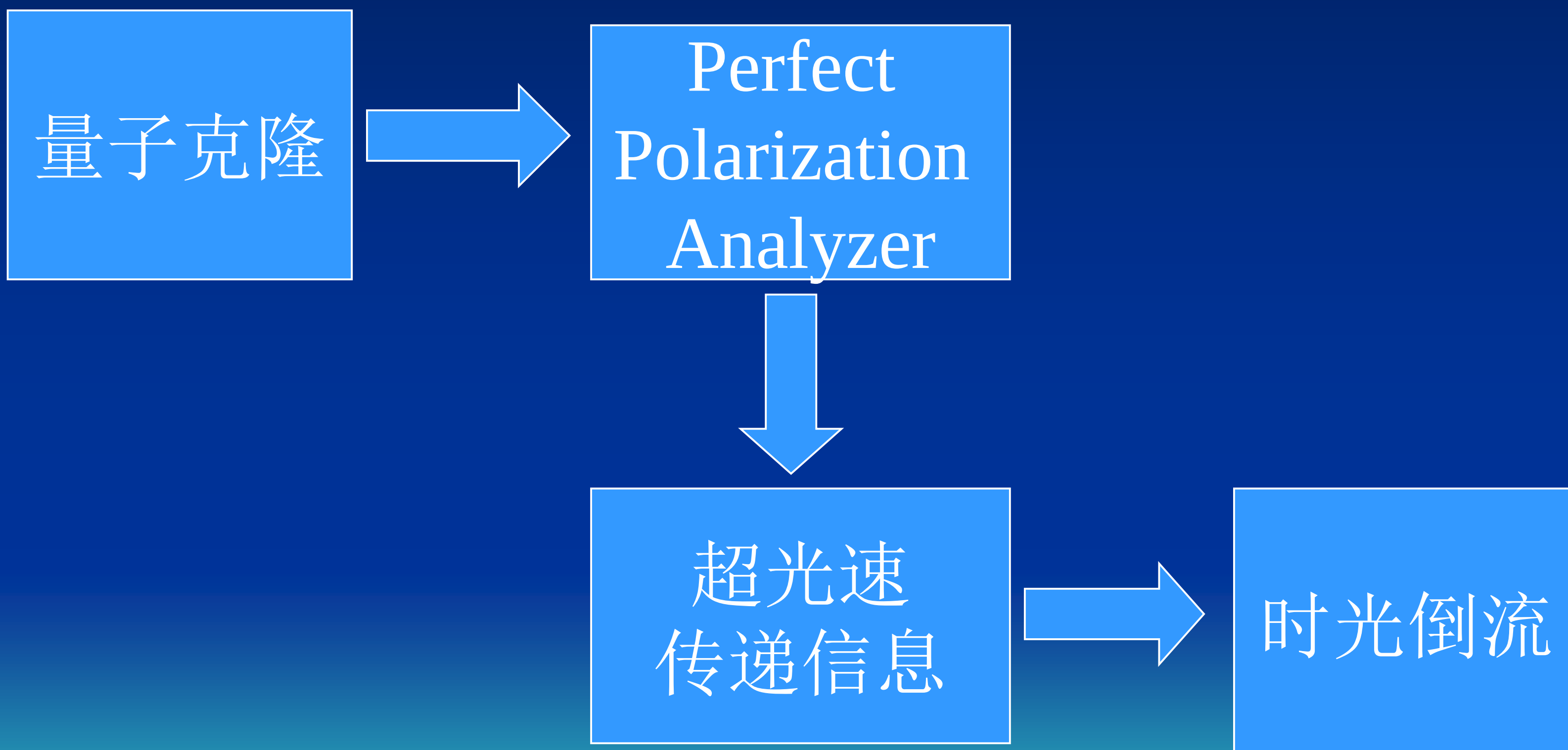
GianCarlo Ghirardi

Department of Theoretical Physics of the University of Trieste, The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy

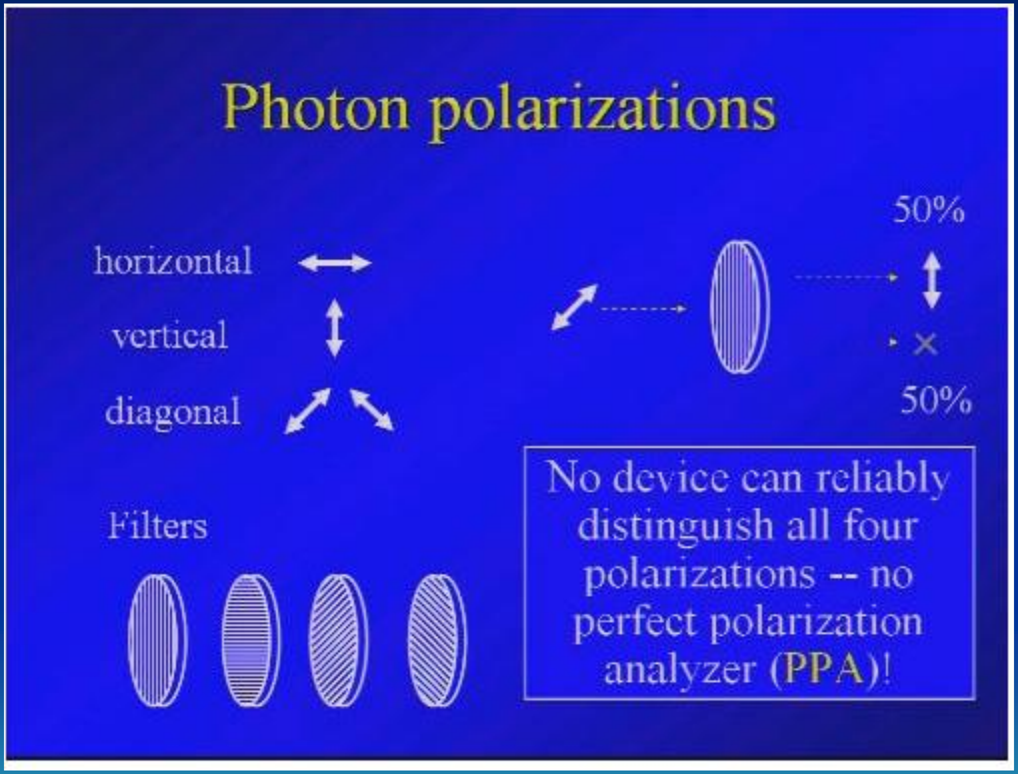
Before leaving this topic I consider it appropriate to recall a marginal fact strictly related to it. In that period I have been involved in refereeing papers claiming that, by resorting to appropriate experimental devices and set-ups, one might actually use the process of WPR for superluminal signalling. One of these papers was due to N Herbert [16] and made use of an hypothetical cloning machine. I spent some days in spotting the flaw in the argument and in so doing I derived and I included in my referee report[4] what became known as the *no cloning theorem* in a form which is practically identical to the one which was presented almost 1 year later in the celebrated paper by Wootters and Zurek [18].

After this parenthesis let us come back to our main theme.

量子克隆: 触犯光速不可逾越原理



态的区分


Photon polarizations
horizontal
vertical
diagonal
50%
50%
Filters
No device can reliably distinguish all four polarizations -- no perfect polarization analyzer (PPA)!

Malus' law (信息观点)

Foundations of Physics, Vol. 32, No. 11, November 2002 (© 2002)

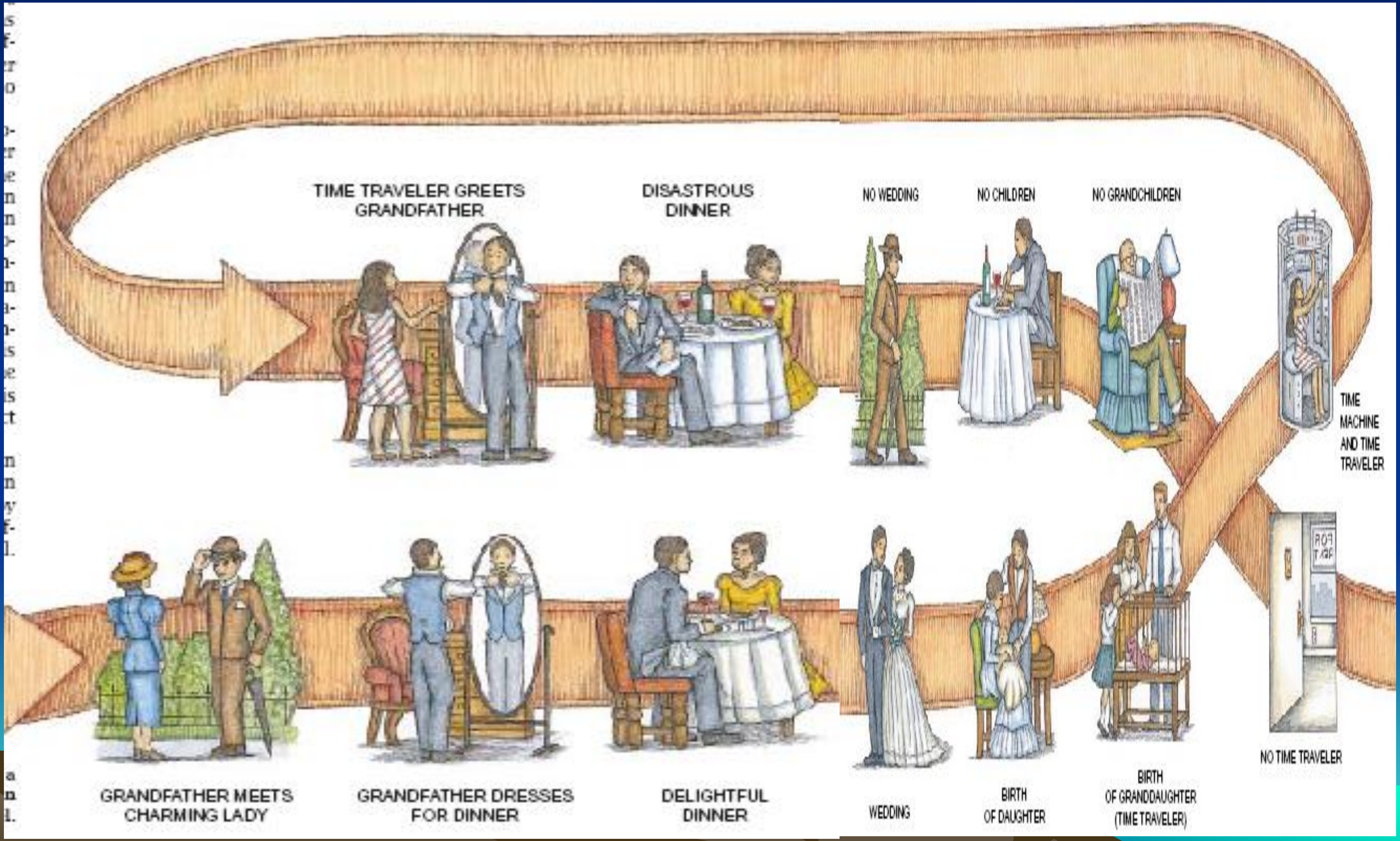
Maximum Shannon Entropy, Minimum Fisher Information, and an Elementary Game

Shunlong Luo[1]

Received March 13, 2002; revised August 22, 2002

We formulate an elementary statistical game which captures the essence of some fundamental quantum experiments such as photon polarization and spin measurement. We explore and compare the significance of the principle of maximum Shannon entropy and the principle of minimum Fisher information in solving such a game. The solution based on the principle of minimum Fisher information coincides with the solution based on an invariance principle, and provides an informational explanation of Malus' law for photon polarization. There is no solution based on the principle of maximum Shannon entropy. The result demonstrates the merits of Fisher information, and the demerits of Shannon entropy, in treating some fundamental quantum problems. It also provides a quantitative example in support of a general philosophy: Nature intends to hide Fisher information, while obeying some simple rules.

量子克隆与时光倒流


TIME TRAVELER GREETS GRANDFATHER
DISASTROUS DINNER
NO WEDDING
NO CHILDREN
NO GRANDCHILDREN
TIME MACHINE AND TIME TRAVELER
NO TIME TRAVELER
GRANDFATHER MEETS CHARMING LADY
GRANDFATHER DRESSES FOR DINNER
DELIGHTFUL DINNER
WEDDING
BIRTH OF DAUGHTER
BIRTH OF GRANDDAUGHTER (TIME TRAVELER)

量子克隆: 触犯Heisenberg测不准原理

Heisenberg测不准原理的一种形式是不可能精确测定一个一般的量子态(量子测量一般总是扰动被测态).

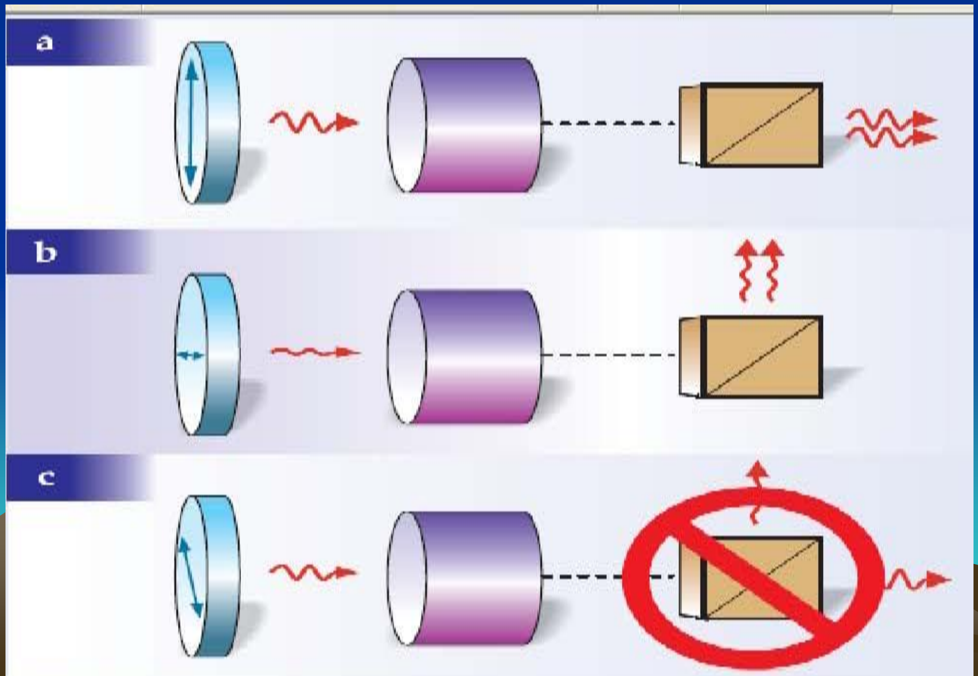
如果存在量子克隆，我们就可以制备很多的被测态，从而可以无限精确地测定它.

量子不可克隆定理

不存在 Hilbert 空间 $H \otimes H$ 上的酉算子 U

满足对任意 $|\varphi\rangle, |a\rangle \in H$，

$$U|\varphi\rangle|a\rangle = e^{i\lambda(\varphi,a)}|\varphi\rangle|\varphi\rangle$$



设 $\{|\psi_i\rangle: i = 1,2,\cdots,n\}$ 是一族纯态，任何两个不正交，$\{\rho_i: i = 1,2,\cdots,n\}$ 是任意态， 则存在物理操作

$$T: |\psi_i\rangle \otimes \rho_i \to |\psi_i\rangle \otimes |\psi_i\rangle, \quad i = 1,2,\cdots,n$$

当且仅当存在物理操作

$$S: \rho_i \to |\psi_i\rangle, \quad i = 1,2,\cdots,n$$

R. Jozsa, A stronger no-cloning theorem, arXiv: quant-ph/0204153v2 (2002)

Heisenberg表象中的量子不可克隆定理

设 $\mathcal{B}$ 是有限维 Hilbert 空间上的一个*-代数，

量子操作

$C: \mathcal{B} \otimes \mathcal{B} \to \mathcal{B}$

称为克隆，如果

$C(B \otimes 1) = C(1 \otimes B) = B, \forall B \in \mathcal{B}.$

在$\mathcal{B}$上存在克隆当且仅当$\mathcal{B}$为交换代数(Abel 代数).

设 H_a, H_b 为 Hilbert 空间，$L(H_a), L(H_b)$为其上观测(自共轭算子)全体. $K: L(H_a) \otimes L(H_b) \to L(H_a)$为量子操作. 则

$$K(1 \otimes B) \subset \{A: K(A \otimes 1) = A\}', \forall B \in L(H_b).$$

G. Lindblad, A general no-cloning theorem,

Lett. Math. Phys. 47, 189 (1999)

Heisenberg 测不准原理的 “No disturbance, no information gain”表述：

设 $\mathcal{A}$, $\mathcal{B}$ 是有限维 Hilbert 空间上的*-代数. 量子操作

$M\colon \mathcal{A}\otimes\mathcal{B}\to\mathcal{A}$ 满足 $M(A\otimes 1)=A,\ \forall A\in\mathcal{A}$,

则 $M(1\otimes B)\in\mathcal{A}\cap\mathcal{A}',\ \forall B\in\mathcal{B}$.

特别地，若 $\mathcal{A}$ 是因子， 则 $M(1\otimes B)=\omega(B)1$.

量子不可用经典来编码

设 $\mathcal{B}_1, \mathcal{B}_2$是有限维 Hilbert 空间上的$*$-代数，且存在

量子操作 (编码) $C: \mathcal{B}_1 \to \mathcal{B}_2$,

量子操作 (编码) $D: \mathcal{B}_2 \to \mathcal{B}_1$,

使得 $DC = 1$.

则 $\mathcal{B}_2$是交换代数 $\Rightarrow$ $\mathcal{B}_1$是交换代数.

既然量子信息不可克隆，

我们只好求其次：

近似克隆

概率克隆

渐近克隆

PHYSICAL REVIEW A VOLUME 54, NUMBER 3 SEPTEMBER 1996

Quantum copying: Beyond the no-cloning theorem

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(Received 5 February 1996)

We analyze the possibility of copying (that is, cloning) arbitrary states of a quantum-mechanical spin-1/2 system. We show that there exists a "universal quantum-copying machine" (i.e., transformation) which approximately copies quantum-mechanical states such that the quality of its output does not depend on the input. We also examine a machine which combines a unitary transformation and a selective measurement to produce good copies of states in the neighborhood of a particular state. We discuss the problem of measurement of the output states. [S1050-2947(96)08408-9]

对量子比特, 最优渐近克隆的保真度为

$$F_{m\rightarrow n} = \frac{m + (m+1)n}{n + (m+1)n}.$$

N. Gisin and S. Massar,

Optimal quantum cloning machines,

Phys. Rev. Lett. 79, 2153 (1997)

概率克隆

Lu-Ming Duan (段路明) and Guang-Can Guo (郭光灿),

Probabilistic cloning and identification of linearly

independent quantum states,

Phys. Rev. Lett. 80, 4999–5002 (1998)

VOLUME 80, NUMBER 22 PHYSICAL REVIEW LETTERS 1 JUNE 1998

Probabilistic Cloning and Identification of Linearly Independent Quantum States

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(Received 19 February 1998)

We construct a probabilistic quantum cloning machine by a general unitary-reduction operation. With a postselection of the measurement results, the machine yields faithful copies of the input states. It is shown that the states secretly chosen from a certain set $S = \{|\Psi_1\rangle, |\Psi_2\rangle, \ldots, |\Psi_n\rangle\}$ can be probabilistically cloned if and only if $|\Psi_1\rangle, |\Psi_2\rangle, \ldots,$ and $|\Psi_n\rangle$ are linearly independent. We derive the best possible cloning efficiencies. Probabilistic cloning has a close connection with the problem of identification of a set of states, which is a type of $n + 1$ outcome measurement on n linearly independent states. The optimal efficiencies for this type of measurement are obtained. [S0031-9007(98)06263-2]

郭光灿 等，概率克隆，1998

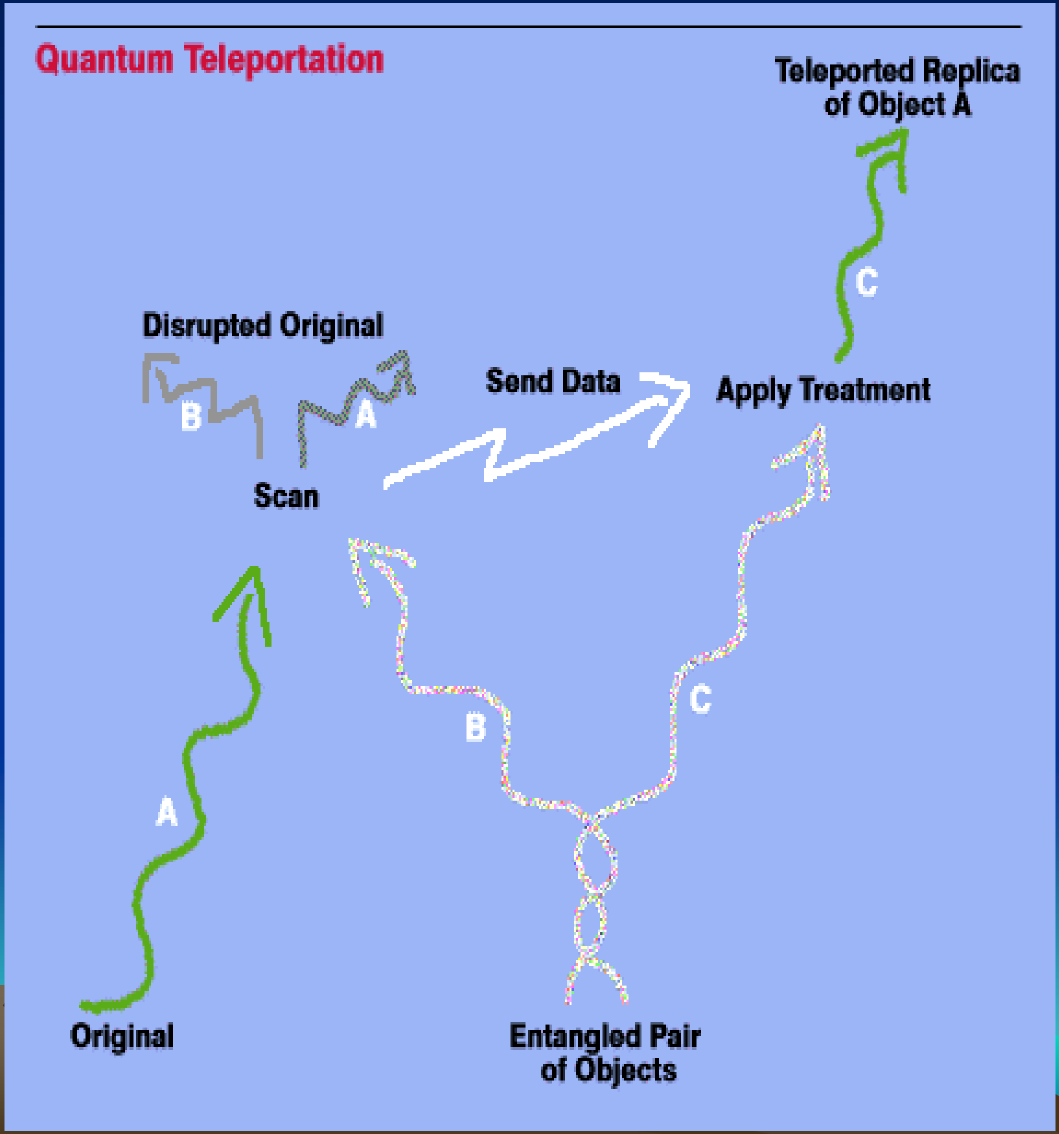
``量子信息技术的基础研究” 2003年国家自然科学二等奖的理论成果：概率克隆，量子避错编码

量子远程传态(quantum teleportation)

量子态不能复制，但远程转移行吗？

可以. 只需破坏原有的系统状态，而把其量子态复制到另一系统上，这就是所谓的量子远程传态.

量子通讯就是基于量子远程传态的.



S. Luo, 从量子不可克隆到波包塌缩，2010

Physics Letters A 374 (2010) 1350–1353

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From quantum no-cloning to wave-packet collapse

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ABSTRACT

One aspect of the longstanding "mystery and weirdness" of wave-packet collapse in quantum mechanics has recently been dissolved by Zurek from an information transfer perspective (Phys. Rev. A 76 (2007) 052110). This result is a significant extension of the original quantum no-cloning theorem (Nature 299 (1982) 802). In this Letter we provide two closely related, but alternative, informational approaches to the orthogonality in wave-packet collapse: The first justifies and refines Zurek's derivation by relaxing the repeatability postulate to a more intuitive and simple one, the second replaces the repeatability postulate by a covariant condition of measuring apparatus. Our derivations illuminate the informational nature of wave-packet collapse.

2. 量子删除

量子信息不可克隆，

那么总可以删除吧？

量子不可删除

不存在酉算子

$$U : |\psi\rangle \otimes |\psi\rangle \otimes |A\rangle \Rightarrow |\psi\rangle \otimes |0\rangle \otimes |A'\rangle$$

使得 $|A'\rangle$ 不依赖于 $|\psi\rangle$

A. K. Pati and S. L. Braunstein, Impossibility of deleting an unknown quantum state, *Nature*, 404 104 (2000)

信息删除与超光速

If it will be possible to delete an unknown quantum state then using two pairs of EPR state we can send signal faster than light. Thus, the no-deleting theorem is in consistent with the no-signalling condition.

3. 量子广播

量子克隆得到的态之间没有关联.

如果允许关联， 则是量子广播.

量子态能广播吗？

一个量子态（通常为混合态）ρ 可用Hilbert空间 H 上的迹为1的非负算子表示.

一个量子操作 E 可用量子态空间 $S(H)$ 上完全正的线性算子表示.

一个量子态 ρ 称为可用可用量子操作

$$E : S(H) \to S(H \otimes H)$$

广播，如果 $E(\rho)$ 的两个边缘态都等于 ρ .

量子关联是信息处理的重要资源

两体量子态的分类

方案 1:　可分/纠缠

方案 2:　经典/量子

关联的分类方案 1：

可分态，纠缠态

R. F. Werner

Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable

Phys. Rev. A, 1989

两体量子态 ρ^{ab} 称为可分态，如果

$$\rho^{ab} = \sum_{\mu} \lambda_{\mu} \rho_{\mu}^{a} \otimes \rho_{\mu}^{b}$$

否则，称为纠缠态.

关联的分类方案 2:
经典关联与量子关联

经典性在测量下不会被扰动.

量子性在测量下会被扰动.

S. Luo, Using measurement-induced disturbance to characterize correlations as classical or quantum *Phys. Rev. A*，2008

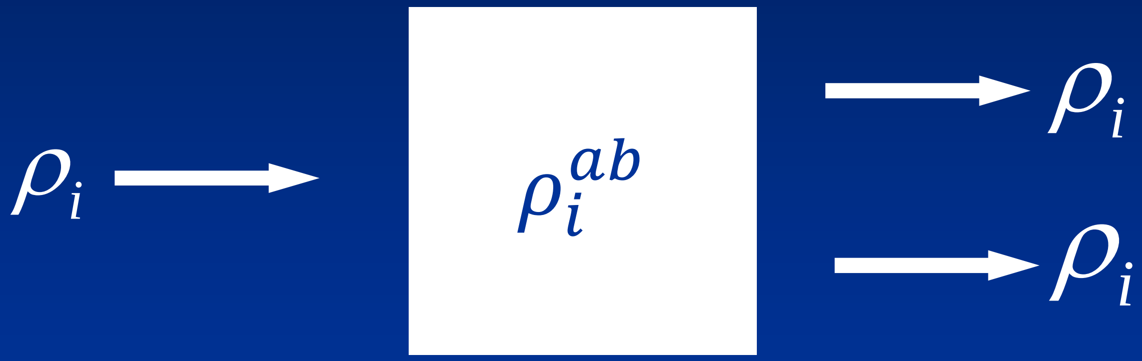
两体量子态 ρ^{ab} 称为经典(关联)的，如果存在局部 von Neumann测量 $\{\Pi_i^a\}$ 和 $\{\Pi_j^b\}$ ，使得该测量不扰动 ρ^{ab} ，即

$$\rho^{ab} = \sum_{i,j} (\Pi_i^a \otimes \Pi_j^b) \rho^{ab} (\Pi_i^a \otimes \Pi_j^b)$$

否则, 称为量子(关联)的.

量子态不可广播定理

一族量子态可同时被广播当且仅当它们可交换.



H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher，

Noncommuting mixes states cannot be broadcast,

Phys. Rev. Lett. 76, 2818 (1996).

量子关联不可广播定理 (双边)

M Piani, P. Horodecki, R. Horodecki, No-local-broadcasting theorem for quantum correlations, Phys. Rev. Lett. 100, 090502 (2008)

两体量子态为经典态当且仅当其中的关联可被局部广播.

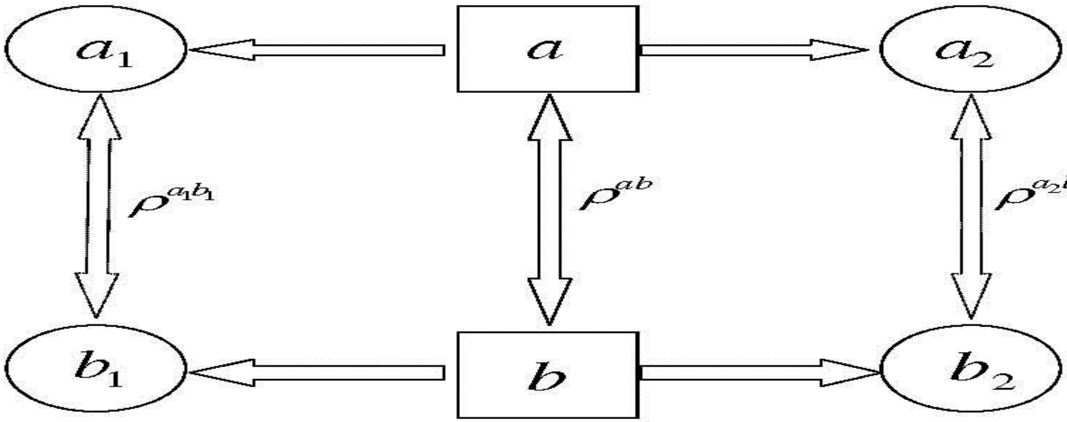
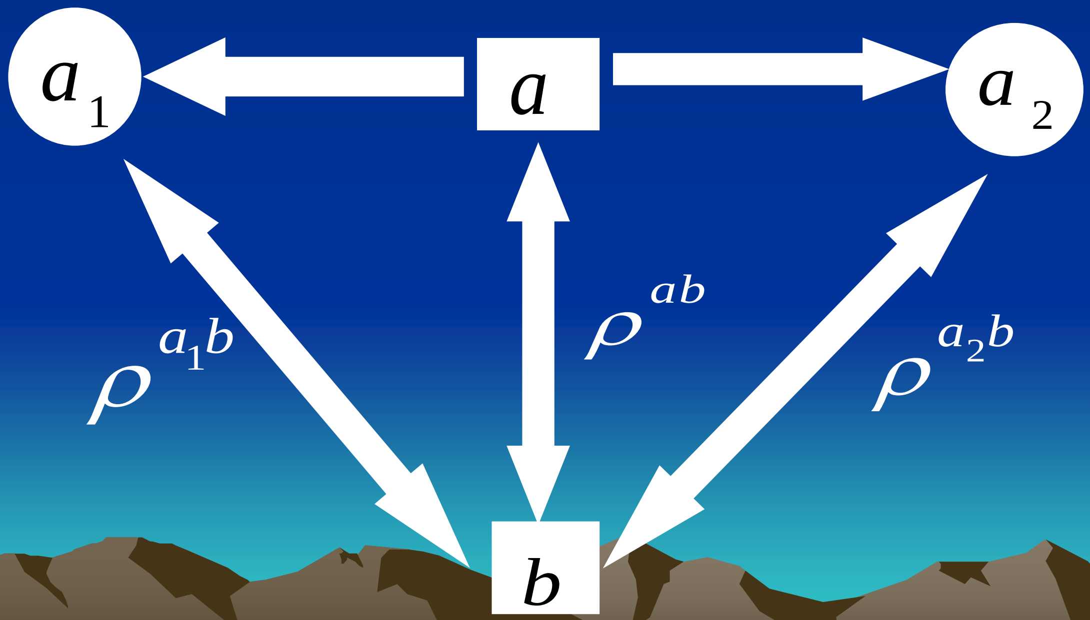


FIG. 1: Local broadcasting of correlations. Local operations $\mathcal{E}^a : \mathcal{S}(H^a) \rightarrow \mathcal{S}(H^{a_1} \otimes H^{a_2})$ and $\mathcal{E}^b : \mathcal{S}(H^b) \rightarrow \mathcal{S}(H^{b_1} \otimes H^{b_2})$ are performed by parties a and b, respectively, with a resulting four-partite state $\rho^{a_1a_2b_1b_2} = \mathcal{E}^a \otimes \mathcal{E}^b(\rho^{ab})$. The two reduced states $\rho^{a_1b_1} = \mathrm{tr}_{a_2b_2}\rho^{a_1a_2b_1b_2}$ and $\rho^{a_2b_2} = \mathrm{tr}_{a_1b_1}\rho^{a_1a_2b_1b_2}$ are expected to reproduce the same amount of correlations as those in ρ^{ab}. The amount of correlations is quantified by the quantum mutual information.

量子关联不可广播定理(单边)

S. Luo and W. Sun, Phys. Rev. A 82, 012338 (2010)

两体量子态ρ^{ab}为经典-量子态当且仅当其可被子系统a局部广播.



量子相对熵的单调性

- Lieb-Ruskai 定理：Von Neumann熵的强次可加性
- 量子相对熵的单调性

E. H. Lieb and M. B. Ruskai, Proof of the strong subadditivity of quantum-mechanical entropy, J. Math. Phys. 14, 1938 (1973)

量子不可广播定理的统一(以下论述互相等价):

- 一族量子态可同时被广播当且仅当它们可交换.
- 两体量子态可被局部广播当且仅当其为经典态.
- 两体量子态 ρ^{ab} 可被系统 a 局部广播当且仅当其为经典−量子态.

S. Luo and W. Sun, Decomposition of bipartite states with applications to quantum no-broadcasting theorems

Phys. Rev. A 82, 012338 (2010)

4. 量子信息

量子信息

- 不可克隆
- 不可删除
- 不可广播
- 不可共享

信息守恒　信息永恒

The no-cloning and the no-deleting theorems point to conservation of quantum information.

A stronger version of the no-cloning theorem and the no-deleting theorem provide permanence to quantum information. To create a copy one must import the information from some part of the universe and to delete one needs to export it to other part of the universe where it will continue to exists.

三体: 单婚性(Monogamy)

经典关联可在多个系统中任意分配.

两个经典概率分布$\{p_{ij}^{ab}\}$和$\{q_{jk}^{bc}\}$只要满足最基本的相容性条件$\{p_j^b\}=\{q_j^b\}$就可粘在一起. 事实上,

$$p_{ijk}^{abc}=p_{i|j}^{a|b}p_j^b q_{k|j}^{c|b}$$

其中 $p_{i|j}^{a|b}=p_{ij}^{ab}/p_j^b$, $q_{j|k}^{b|c}=q_{jk}^{bc}/q_j^b$

为条件概率.

在量子情形, 情况完全不同.

经典世界

一个经典粒子在某一时刻只能在一个地方.

经典关联很易共存.

量子世界

一个量子粒子可同时在

多个地方.

量子关联很难共存.

量子关联不能共享:

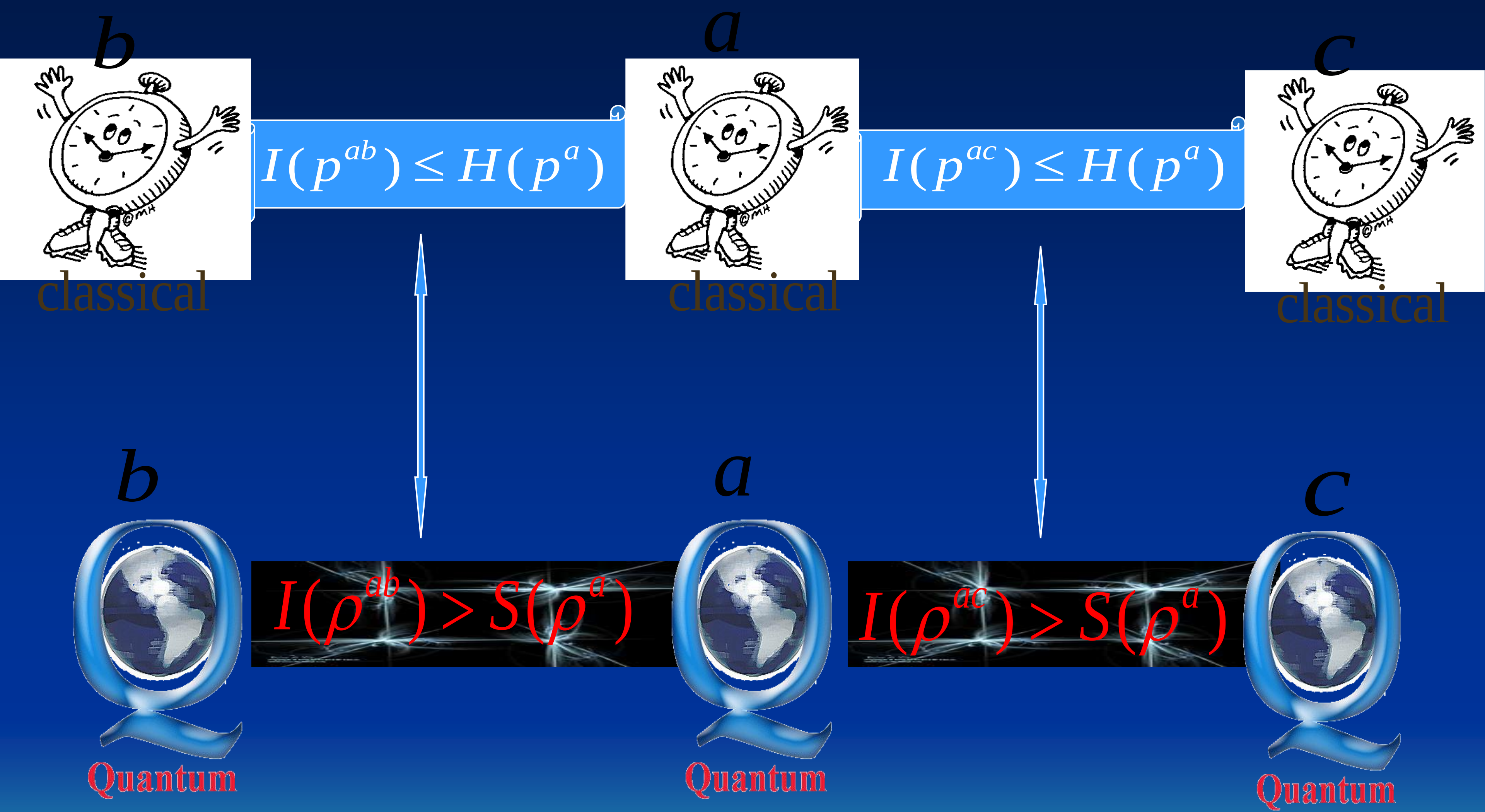
排他性，单婚性

考虑任意三体量子态 ρ^{abc}， 如果a与b有很强的纠缠(量子关联), 则 a 与c就不能有很强的纠缠(量子关联).

考虑任意三体量子态ρ^{abc}，如果系统a与系统b有完全关联，则a不能与其它任何别的系统建立任何量子关联.

S. Luo and W. Sun,

Separability and entanglement in tripartite systems, *Theoretical and Mathematical Physics*, 160, 1315-1322 (2009)



$$I(\rho^{ab}) + I(\rho^{ac}) \leq 2S(\rho^{a})$$

Koashi-Winter 单婚性

经典关联与纠缠的单婚性

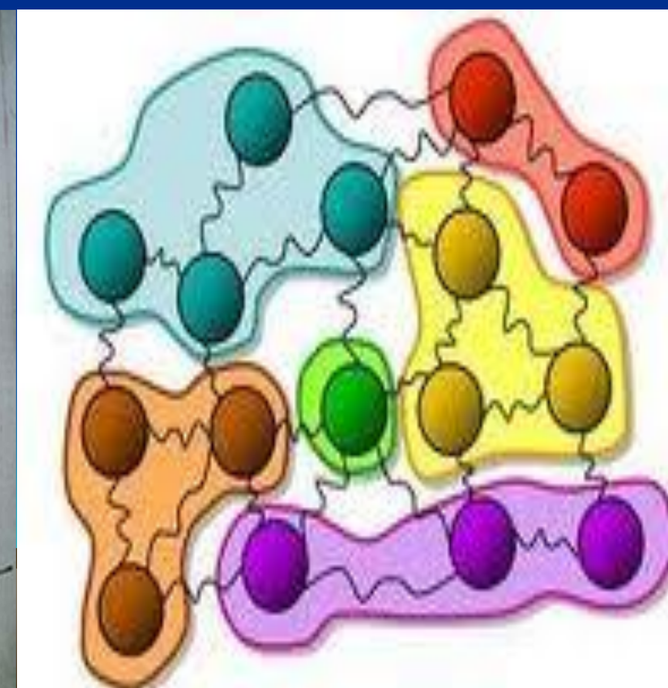
$$C(\rho^{ab}) + E(\rho^{ac}) = S(\rho^{a})$$

Marginal 问题

如何将多个两体关联粘在一起?

各种关联如何共存？

什么是允许的?什么是禁止的?



量子不可克隆定理的前世今生

- 前世

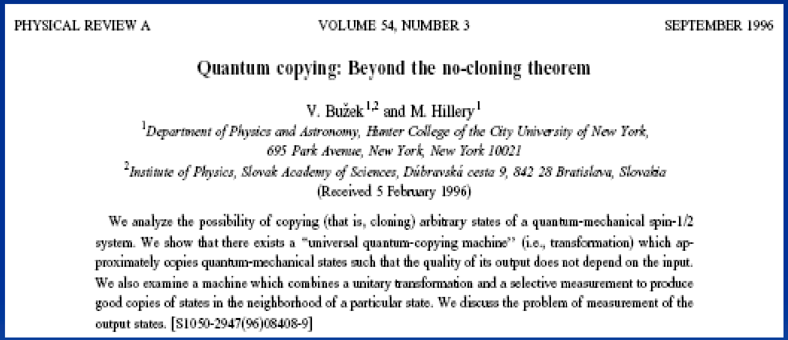
1970年代，已在无意中出生，可惜生不逢时，无人搭理， 湮灭几无迹！

1980年代，再次由嬉皮士催生 (多篇文章发表在 Nature 上宣告之)，似乎还是生不逢时，除了起着否定了一个嬉皮士的奇思怪想，别无他用，还是几乎无人搭理！ (GoogleScholar ``Quantum no-cloning”, 1900-1995, 得27条)

量子不可克隆定理的前世今生

- 今生

1996年，时来运转，以下文章的发表(被引次数：1111)，引起了量子不可克隆定理的研究和应用的热潮！ (Google Scholar ``Quantum no-cloning”, 1995--,得6860条)


PHYSICAL REVIEW A VOLUME 54, NUMBER 3 SEPTEMBER 1996

Quantum copying: Beyond the no-cloning theorem

V. Bužek[1,2] and M. Hillery[1]

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(Received 5 February 1996)

We analyze the possibility of copying (that is, cloning) arbitrary states of a quantum-mechanical spin-1/2 system. We show that there exists a "universal quantum-copying machine" (i.e., transformation) which approximately copies quantum-mechanical states such that the quality of its output does not depend on the input. We also examine a machine which combines a unitary transformation and a selective measurement to produce good copies of states in the neighborhood of a particular state. We discuss the problem of measurement of the output states. [S1050-2947(96)08408-9]

究其原因： 其一是量子信息的兴起，其二是以上文章开创了量子不可克隆的量化研究.

量子克隆具有

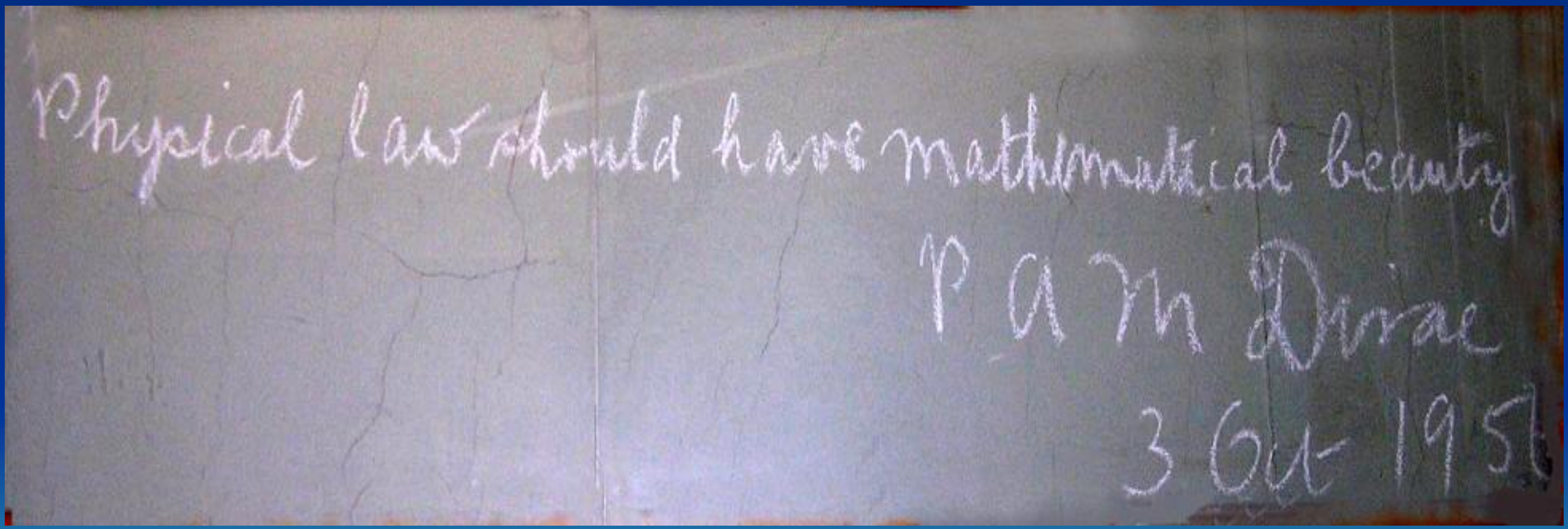
- 深刻普适的物理内涵
- 优美丰富的数学结构

为研究以下基本问题提供了新思路

- 量子测量，退相干，Quantum-to-classical transition
- 信息守恒原理
- 克隆的数学理论

量子不可克隆的很多刻画都具有数学美

Dirac 1951 年在莫斯科大学演讲时被问及他的物理哲学, 他在黑板上写道(这黑板至今仍被保存着):



一个物理定律必须具有数学美.

华罗庚应用数学讲座

Hua Lookeng Distinguished Lecture

Forging the Culture of Quantum Information Science

Charles H. Bennett

美国国家科学院院士

美国IBM T.J.Watson 研究中心IBM Fellow

量子信息的奠基者

Abstract: Physicists, mathematicians and engineers, guided by what has worked well in their respective disciplines, have historically developed different scientific tastes, different notions of what constitutes an interesting, well-posed problem or an adequate solution. While this has led to some frustrating misunderstandings, it has invigorated the theory of communication and computation, enabling it to outgrow its brash beginnings with Turing, Shannon and von Neumann, and develop a coherent scientific taste of its own, domesticating ideas from thermodynamics and quantum mechanics that physicists had mistakenly thought belonged solely to their field, to better formalize the core concepts of communication and computation.

时间：2017年11月12日(周日) 9:00-9:50

地点：中国科学院数学与系统科学研究院 南楼 204 室

主办单位：中国科学院数学与系统科学研究院应用数学研究所

联系电话：010-82541756